

Entropy-driven formation of large icosahedral colloidal clusters by spherical confinement

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Icosahedral symmetry, which is not compatible with truly long-range order, can be found in many systems, such as liquids, glasses, atomic clusters, quasicrystals and virus-capsids^{1–12}. To obtain arrangements with a high degree of icosahedral order from tens of particles or more, interparticle attractive interactions are considered to be essential^{1,3,6–12}. Here, we report that entropy and spherical confinement suffice for the formation of icosahedral clusters consisting of up to 100,000 particles. Specifically, by using real-space measurements on nanometre- and micrometre-sized colloids, as well as computer simulations, we show that tens of thousands of hard spheres compressed under spherical confinement spontaneously crystallize into icosahedral clusters that are entropically favoured over the bulk face-centred cubic crystal structure^{13,14}. Our findings provide insights into the interplay between confinement and crystallization and into how these are connected to the formation of icosahedral structures.

More than half a century has passed since Sir Charles Frank first proposed that the most favourable local structures in simple liquids have short-ranged icosahedral symmetry¹². Such ordering occurs when 12 particles are arranged around a central particle at the vertices of an icosahedron. This icosahedron tends to minimize the short-ranged (Lennard-Jones-like) attractive interactions typically present in atomic systems, and has been shown to be entropically favourable as well^{15,16}. However, its five-fold symmetry is incommensurate with long-range positional order, thereby acting as an obstacle to the formation of crystals on a larger scale. So, even though a typical liquid often contains many local icosahedral centres^{1–3,12}, they rarely grow out into a crystal⁴. In the case of hard spheres, which do not attract and instead interact solely by excluded volume, (distorted) icosahedral order has been observed in the fluid phase⁵, in glasses⁵ and in growing crystal nuclei during crystallization¹⁷. The thermodynamically stable crystalline phase for hard spheres is the face-centred cubic (FCC) crystal, which maximizes the entropy at high densities^{13,14}. This arrangement of spheres has the densest possible packing ($\Phi = 0.74$) at infinite pressure, and does not exhibit any five-fold symmetry^{13,14}. The incompatibility of the locally favourable icosahedral symmetry with long-range three-dimensional (3D) order raises immediately the question over what length scales icosahedral order can be extended.

A dense non-crystalline packing of spheres featuring global icosahedral order was theoretically proposed by Mackay in 1962⁶. The Mackay structure (which will be described in more detail below) has been observed and studied theoretically for atomic and molecular clusters^{9,10,18,19} and clusters of nanoparticles^{11,20–22}. In these

cases, the formation of icosahedral structures is mainly attributed to energetic interactions^{7,8,11} in conjunction with kinetic effects²⁰, hierarchical self-assembly¹⁸, or intricate growth mechanisms^{21,22}. Furthermore, Mackay clusters of Lennard-Jones particles have been shown to correspond to minima in the free-energy landscape²³. In contrast, in hard-sphere systems attractions are not present and finite-size clusters can be obtained by means of confinement. It is known from theory, simulations and experiments that confinement can change the equilibrium crystal structure of colloidal spheres markedly from that in the bulk, but usually only on small length scales^{24,25}. Here we show by experiments and simulations that spherical confinement gives rise to purely entropy-driven icosahedral symmetry in the equilibrium phase of even very large numbers of hard spheres.

To experimentally study the behaviour of colloidal spheres in spherical confinement, we synthesized cobalt iron oxide nanoparticles²⁶ with a core diameter of 6.0 ± 0.29 nm (9 nm effective diameter due to interdigitating oleic acid ligands, and an effective polydispersity of 3.2%; see Supplementary Fig. 1) dispersed in a suitable (apolar) solvent. This dispersion was emulsified into an oil-in-water emulsion. Subsequently, we evaporated the solvent in the suspended emulsion droplets, causing the packing fraction of the nanoparticles in the droplets to increase slowly, which eventually caused crystallization of the nanoparticles. The same process was used for fluorescently labelled micrometre-sized core-shell silica colloids²⁷, simply called ‘colloids’ in the remainder of the paper, with a diameter of 1.32 ± 0.039 μ m (and a polydispersity of 1.7%; see Supplementary Fig. 2), as well as for core-shell semiconductor nanoparticles with a core diameter of 12.4 ± 1.0 nm (14.5 nm effective diameter, and an effective polydispersity of 6.9%; results shown in Supplementary Figs 1 and 4). The colloids were dispersed in a water-in-oil emulsion instead of an oil-in-water emulsion. In all cases, the evaporation times were much longer than the diffusional equilibration time of the colloidal systems. The resulting clusters obtained via this self-assembly process, which we denote as ‘supraparticles’, were almost entirely crystalline. To illustrate this, in Fig. 1 we show secondary electron scanning transmission electron microscopy (SE-STEM) images of typical supraparticles of cobalt iron oxide nanoparticles. Remarkably, we observed three different types of crystalline packing, depending on the cluster size. Figure 1a shows a 105 nm diameter supraparticle that exhibits icosahedral symmetry, closely resembling the packing proposed by Mackay⁶ and analogous to the clusters of gold nanoparticles formed by a similar process in ref. 11. As we will show below, it consists of twenty deformed FCC ordered tetrahedral domains that share a

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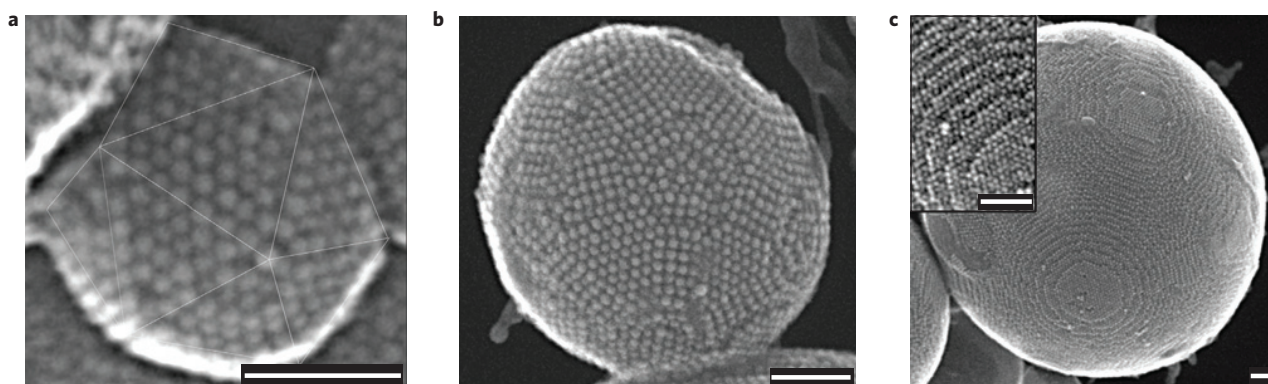


Figure 1 | Secondary electron scanning transmission electron microscopy (SE-STEM) images of typical supraparticles containing cobalt iron oxide nanoparticles. **a**, Supraparticle with a diameter of 105 nm with Mackay icosahedral symmetry, as indicated by the thin lines. **b**, 216 nm supraparticle with anti-Mackay rhombicosidodecahedral structure. **c**, 734 nm supraparticle consisting of a single face-centred cubic (FCC) crystal domain. Inset: a magnified view of the step edges of the FCC supraparticle. All scale bars are 50 nm.

particle in the centre, such that the (111) adjacent tetrahedral faces form twinning planes. The resulting supraparticle is a multiply-twinned crystal with five-fold symmetry, and has the shape of an icosahedron with 20 triangular (111) facets at the surface. For larger supraparticles, however, we find a different surface, shown in Fig. 1b, where a 216 nm supraparticle is depicted. The surface particles are arranged in a rhombicosidodecahedral geometry consisting of twelve pentagonal faces, twenty triangular faces and thirty rectangular faces. This surface-reconstructed icosahedral structure belongs to the class of the anti-Mackay icosahedra, as found for instance in clusters of argon²⁸ and lead⁹ atoms, as well as in clusters of gold nanoparticles¹¹. More precisely, these structures have a Mackay icosahedral core but a different surface termination, as the triangular facets meet yet another set of twinning planes near the surface⁷. Finally, for sufficiently large cluster sizes, the supraparticles consist of a single FCC domain of nanoparticles, as expected for systems approaching the bulk limit. Figure 1c shows an example of such a non-icosahedral FCC supraparticle with a diameter of 734 nm. In this case, the surface presents the typical step edges of an FCC crystal confined to a sphere. A further image of an FCC cluster is shown in Supplementary Fig. 3.

To confirm that the internal structure of our clusters corresponds to (anti-)Mackay icosahedra, we obtained the 3D-coordinates of the individual particles in several clusters. In particular, we extracted the positions of the nanoparticles from electron tomography images²⁹ (see Supplementary Movies 1 and 2), and the colloids from confocal microscopy images³⁰, using particle tracking software (see Supplementary Methods and Supplementary Figs 6 and 7 for more details). Figure 2 shows the structure of typical clusters, exhibiting icosahedral symmetry formed from both nanoparticles and colloids, with different colours indicating different crystal domains as identified by a bond-orientational order parameter². The characteristic five-fold symmetries of Mackay icosahedra are clearly visible in the interior of all clusters (top row of Fig. 2). The larger supraparticles also showed anti-Mackay icosahedral surface terminations (bottom row of Fig. 2). Cross-sections of the internal structure are visualized in Supplementary Figs 8 and 9, and in the Supplementary Data (WebGL). Note that the assemblies made from the colloids resulted in only partially ordered clusters, where roughly half of the cluster resembled a Mackay or an anti-Mackay icosahedron, and the other half consisted of disordered colloidal particles. This was observed in all the colloidal clusters we examined, and is almost certainly due to gravitational sedimentation of the colloids within their emulsion droplets, as well as to the sedimentation and deformation of the droplets themselves.

To study the size dependence of the cluster symmetry in more detail, we examined 121 supraparticles containing between approximately 430 and 500,000 nanoparticles (with a resulting supraparticle diameter between 75 nm and 785 nm). We then determined the cluster structure from the SE-STEM images of the particles at the surface (as in Fig. 1) after having determined, for a much more limited number of supraparticles, that this was compatible with the interior structure. In the estimation of the number of nanoparticles, we assumed that the volume fraction within each supraparticle corresponded to that of hard spheres at close packing ($\Phi \approx 0.74$). As shown in Fig. 3, the transition from a Mackay icosahedron to an anti-Mackay rhombicosidodecahedron was found to be between 1,000 and 3,000 nanoparticles per supraparticle, and the transition to purely FCC ordering occurred between 25,000 and 90,000 nanoparticles. Clusters with more than 90,000 nanoparticles exhibited solely FCC ordering. We find the same three structures for micrometre-sized colloids and the core-shell semiconductor nanoparticles (Supplementary Figs 4 and 5).

The clear emergence of icosahedral symmetry in these fairly different experimental systems strongly indicates that this behaviour is not interaction specific, and poses the question of whether the formation of icosahedral clusters could be purely entropy-driven. To explore this hypothesis, we performed event-driven molecular dynamics simulations of hard spheres in a hard spherical confinement, and slowly shrunk the confining sphere, thereby mimicking the evaporation process. As shown in Fig. 2, our simulations clearly demonstrate that icosahedral ordering arises spontaneously for hard spheres in this confinement, and reveal striking agreement with the experimentally observed structures. Moreover, we studied the cluster-size dependence in simulations, and found that the transitions from Mackay to anti-Mackay to FCC ordering approximately matched those shown in Fig. 3 (Supplementary Fig. 10). In further simulations, we confirmed that the structure did not change significantly when changing the shrinking rate (corresponding to the evaporation rate) of the spherical confinement, assuming that the shrinking was sufficiently slow such that the system remained in quasi-equilibrium during the self-assembly process. Similarly, adding a short-ranged repulsion between the spheres and the confining wall, to model surface tension effects, gives rise to more faceted supraparticles, as the confinement is less severe, but does not affect the icosahedral symmetry (Supplementary Fig. 10). Furthermore, we obtained similar results in the case of an attractive wall-particle interaction (Supplementary Fig. 11).

To examine the thermodynamic stability of the icosahedral clusters, we performed free-energy calculations³¹ on all the

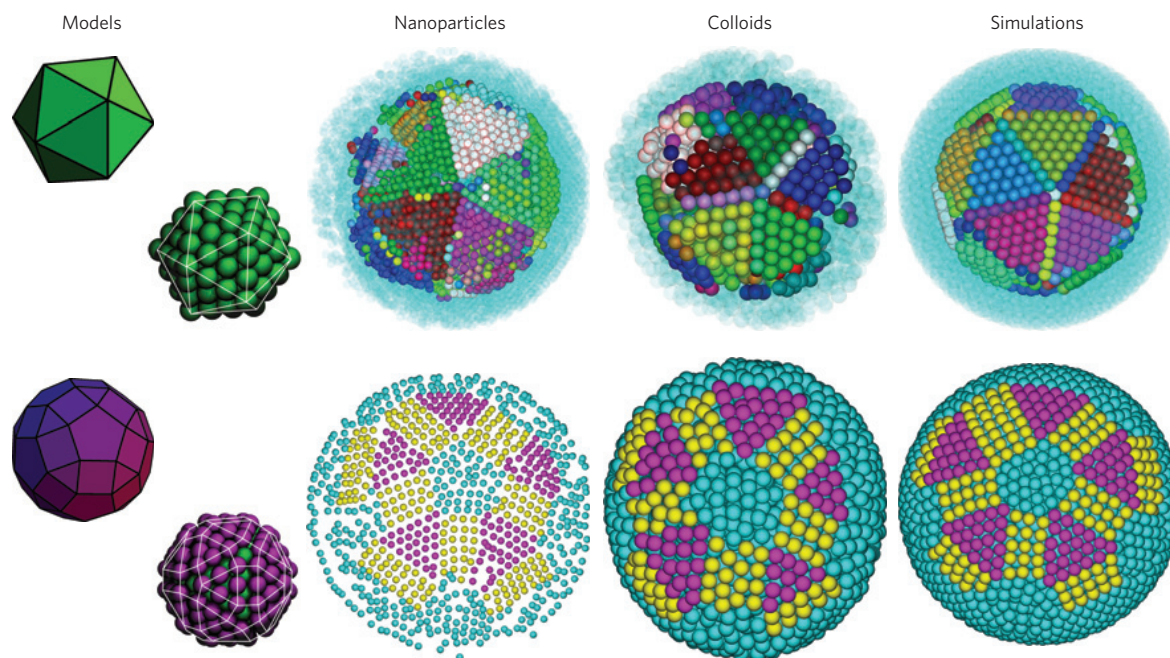


Figure 2 | Core and surface structure of the icosahedral clusters. Core (top) and surface termination (bottom) of large icosahedral supraparticles resulting from the self-assembly of spherically confined colloidal spheres. The first column depicts models of the corresponding polyhedra and their associated ideal sphere packings. The other columns contain typical examples of clusters in (from left to right) experimental systems of nanoparticles ($N \approx 12,000$), experimental systems of micrometre-sized colloids ($N \approx 3,000$), and simulations of hard spheres ($N = 6,000$). Note that in the top row particles belonging to the outer layers have been made transparent so that the core, which exhibits Mackay icosahedral symmetry, is readily visible. Crystalline domains are indicated by different colours. In the bottom row, the particles are coloured as a guide to the eye to highlight the anti-Mackay rhombicosidodecahedral symmetry of the clusters. See Supplementary Methods for details on particle tracking, domain identification and image processing.

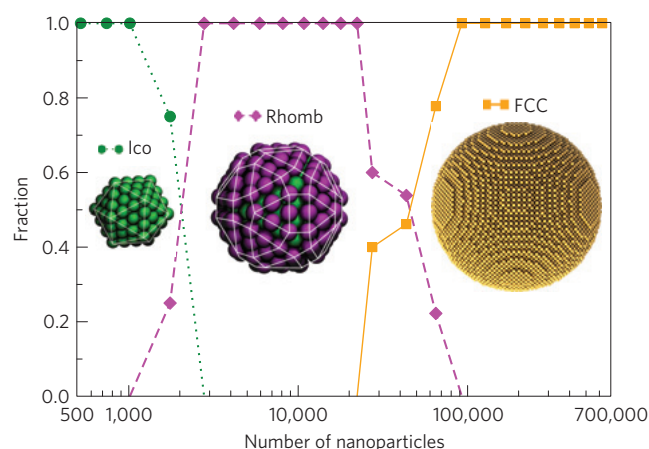


Figure 3 | Size dependence of the cluster structure. Structural transition from a Mackay icosahedron (Ico) to an anti-Mackay rhombicosidodecahedron (Rhomb) to a face-centred cubic (FCC) cluster, as observed for supraparticles consisting of nanoparticles. The fraction of structures, based on 121 supraparticles, is plotted as a function of the number of nanoparticles per supraparticle. 14 icosahedra, 63 rhombicosidodecahedra and 44 FCC clusters were observed.

structures that were observed in experiments and simulations—that is, FCC and Mackay icosahedral clusters. We compared the free energy of the two types of clusters containing between 1,500 and 4,000 particles. We found the icosahedral cluster to be more stable than the FCC for packing fractions close to melting, with free-energy differences of $0.03 \pm 0.01 k_B T$ per particle (Supplementary Table 1). Hence, we conclude that the presence of icosahedral ordering is purely entropy-driven, and is not simply a kinetic,

but rather a genuine equilibrium effect—thus explaining the high reproducibility of the icosahedral clusters. Furthermore, we remark that the free-energy differences due to entropy as reported in Supplementary Table 1 are of the same order as the potential-energy difference between icosahedral and FCC Lennard-Jones clusters¹⁹. Our results thus show that entropy could play an important role in determining the cluster structure even in cases where the clusters are stabilized by energetic interactions. We note here that even for small clusters the FCC structure should become stable for sufficiently high densities, as the maximum packing fraction of icosahedral clusters is $\Phi \approx 0.69$, which is significantly lower than that of a close-packed FCC cluster⁶.

The evolution of the crystal structure, and thus the nucleation and growth mechanism, can also be studied directly in the simulations by identifying the crystalline particles in the system using a bond-orientational order parameter². In Fig. 4 we examine the evolution of a system which forms an anti-Mackay rhombicosidodecahedron. Using the cone algorithm¹⁰, we determine the fraction of crystalline particles in the layers near the confining wall and that of the interior as a function of packing fraction. Note that the packing fraction increases with time during the simulations, so an increase in packing fraction can also be considered as moving forward in time. We clearly see that the crystallization starts near the spherical interface, initially forming approximately two or three layers. When the packing fraction reaches approximately $\Phi = 0.52$, part of the interior starts to crystallize, growing on the already present crystalline domains in the surface layers and proceeding inwards. Furthermore, as the interior starts to crystallize, the surface layer becomes less crystalline (the decrease of the red curve in Fig. 4). Between the volume fractions $\Phi = 0.52$ and $\Phi = 0.53$, the interior completely crystallizes, forming the tetrahedral domains associated with the Mackay icosahedron. It is worth noting that this process is very

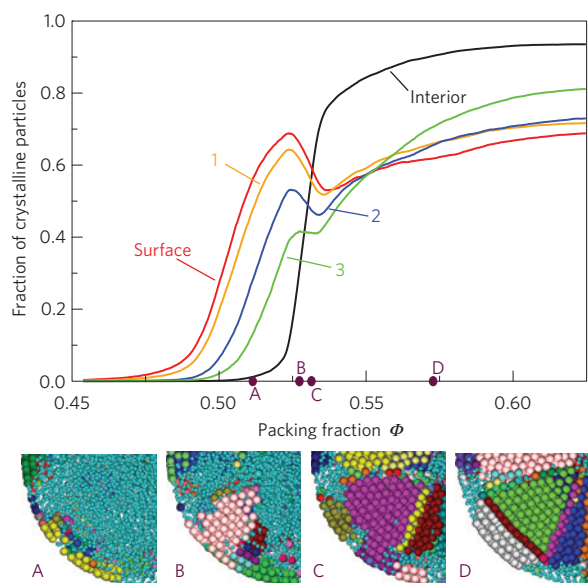


Figure 4 | Crystallization process of icosahedral clusters. Crystallization process studied by event-driven molecular dynamics simulations of 16,000 hard spheres in a shrinking spherical confinement. Top: fraction of crystalline particles in the surface layer, the first three layers and the interior as a function of the packing fraction. Note that as a result of the shrinking confinement, the packing fraction slowly increases over time. Bottom, from left to right: typical configurations from simulations at different packing fractions ($\Phi_A \approx 0.511$, $\Phi_B \approx 0.527$, $\Phi_C \approx 0.531$, $\Phi_D \approx 0.572$) as indicated on the graph. Crystalline domains are indicated with different colours. Fluid-like particles are shown with a smaller diameter and are coloured light-blue.

dynamic, as the domains can crystallize and melt several times before the system fully crystallizes into 20 tetrahedral FCC units (Supplementary Movies 3 and 4). On further increasing the packing fraction, the surface layers recrystallize into the anti-Mackay surface termination. This crystallization process is reminiscent of the nucleation of hard spheres near hard spherical seeds³². In this case, crystal nucleation starts on the surface of the seed, which then grows out into the bulk, whereas the crystal near the surface melts. In contrast, our simulations of smaller numbers of confined hard spheres, which self-assemble into Mackay icosahedra, did not exhibit melting or recrystallization (Supplementary Fig. 12).

In conclusion, we showed that entropy and spherical confinement alone are sufficient for the formation of stable icosahedral clusters. Our simulations clearly demonstrate that energetic interactions between the particles are not required for icosahedral order. Interestingly, this also provides new insights regarding the results reported in ref. 11: whereas the authors of that study attributed the formation of icosahedral clusters of gold nanoparticles to energetic interactions, it is now clear that entropic contributions should not be overlooked. Furthermore, as already argued in ref. 11, we clearly find that the interaction between the particles and the interface does not seem to play an important role in this self-assembly process. In fact, our simulations show similar results regardless of whether the interface–particle interaction is attractive, hard, or repulsive. Our results also provide an interesting contrast to the icosahedral order observed in much smaller ($N = 12$) clusters of particles³³. In such systems, it was argued that the structure of the clusters was mainly determined by capillary forces during the final stages of the evaporation process, resulting in clusters that minimized the second moment of the mass distribution³⁴. Here, we find that the spherical confinement provided by the surface tension of emulsion droplets is sufficient

for the formation of large icosahedral clusters that minimize the free energy. As a consequence, we speculate that our results obtained with emulsions will probably be similar to those obtained in the more general case of the slow drying of droplets of colloidal dispersions. It remains for future work to analyse to what extent the entropy contributes in stabilizing clusters of particles with long-ranged attractions, if it is possible to increase the size range of equilibrium icosahedral supraparticles using particles with long-range interactions, and how the present results can be extended to (hard) binary systems.

The fact that spherical confinement can stabilize structures that are fully incommensurate with that of the bulk, suggests new ways of designing small crystals with unusual symmetries that may be beneficial for optical or other applications. This opens up a new avenue for the self-assembly of novel structures by combining the vast array of colloidal building blocks available at present (for example, rods, dumbbells), and mixtures thereof. Furthermore, the resulting supraparticles themselves can in turn self-assemble³⁵, resulting in hierarchical structures with new functionalities added at different length scales—for example, with plasmonic properties at the nanoscale and photonic properties at the micrometre scale.

Methods

Experiments. For the nanoparticle supraparticles, 6.0 nm (9 nm effective, including the oleic acid ligands) cobalt iron oxide nanoparticles²⁶ were dispersed in cyclohexane and mixed with water containing sodium dodecyl sulfate and dextran. The mixture was sheared at a shear rate of $1.56 \times 10^{-5} \text{ s}^{-1}$ and the resulting oil-in-water emulsion was heated at 68°C for four hours. The resulting supraparticles were washed by centrifugation and freeze-dried on a carbon-coated TEM grid. The supraparticles were analysed using electron tomography²⁹. For the colloidal supraparticles, $1.32 \mu\text{m}$ fluorescent core-shell silica colloids²⁷ were dispersed in de-ionized water. The suspension was added to hexadecane containing the surfactant Span 80 and shaken. The resulting water-in-oil emulsion was left to evaporate, and the obtained supraparticles were analysed using quantitative 3D confocal microscopy³⁰. To identify the crystalline domains, a cluster criterion based on local bond-order parameters² was used. For a detailed description of the chemicals and equipment used, see the Supplementary Methods.

Simulations. We perform event-driven molecular dynamics (EDMD) simulations of hard spheres with a diameter σ confined in a spherical cavity. To model the spherical confinement, we use three types of external potential that act on the hard spheres: an impenetrable hard spherical wall of radius R , a soft repulsive wall–particle potential with an interaction range $\leq 2\sigma$, and an attractive wall–particle potential with an interaction range of σ . To mimic the solvent evaporation of the emulsion droplets, the radius of the spherical cavity is slowly reduced at a constant compression speed $v \leq 10^{-4} \sigma/\tau$, where $\tau = \sqrt{m\sigma^2/k_B T}$ is the EDMD time unit, m the mass of a particle, k_B the Boltzmann constant and T the temperature. We use an Andersen thermostat to keep the temperature of the system constant during the simulations. To identify the crystalline particles, we used the same procedure as for the experimental data. In addition, we employed the cone algorithm¹⁰ to study the crystallization mechanism.

We calculate the free energy of both FCC clusters and icosahedral clusters using a three-step thermodynamic integration method proposed in ref. 31. In particular, this method allows us to calculate the free-energy difference between our system—for example, spherically confined hard spheres—and a reference system for which the free energy is known exactly. The reference system we use consists of non-interacting particles attached, via a linear well potential, to lattice sites. The lattice sites are taken from representative configurations of the system featuring the desired cluster symmetry. In the first integration step, the coupling between the particles and their respective lattice sites is slowly turned on. To accurately perform this integration, we sampled more than 300 values of the coupling constant, and fit the results with an Akima spline before performing the integration. The other two integration steps consist of slowly switching off the hard interactions in the model—namely, the particle–particle and particle–wall interactions. These last two integrations were done using a standard 20-point Gauss–Legendre integration scheme.

More simulation details are reported in the Supplementary Methods.

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Author contributions

A.I. and A.v.B. initiated the experimental part of the project. B.d.N. and D.J.G. performed the experiments under the supervision of A.I. and A.v.B. B.d.N. and J.D.M. performed the electron microscopy analysis. S.D. and F.S. carried out the simulations under the supervision of L.F. and M.D. B.d.N., S.D., F.S., L.F., M.D. and A.v.B. co-wrote the paper. All authors analysed and discussed results.

Additional information

Supplementary information is available in the [online version of the paper](#). Reprints and permissions information is available online at www.nature.com/reprints. Correspondence and requests for materials should be addressed to A.v.B. or M.D.

Competing financial interests

The authors declare no competing financial interests.

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Supplementary information

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Supplementary Methods

Chemicals

All chemicals were used as received. Chemicals used were dextran from *Leuconostoc mesenteroides* (Sigma Aldrich, mol. wt. 1.500.000 - 2.800.000), cyclohexane (Sigma Aldrich, $\geq 99.8\%$), sodium dodecyl sulfate (SDS, Sigma Aldrich, $\geq 99.0\%$), toluene (Acros, $\geq 99.7\%$), 1-hexadecene ($\geq 99\%$), hexadecane (Sigma Aldrich, $\geq 99\%$), Span 80 (Sigma Aldrich), oleic acid (Sigma Aldrich, $\geq 90\%$), Fluorescein isothiocyanate (Sigma Aldrich, isomer I, FITC, $\geq 90\%$), ethanol (Baker Analyzed, $\geq 99.9\%$, absolute). For de-ionized water a Millipore Direct-Q UV3 reverse osmosis filter apparatus was used ($18\text{ M}\Omega$ at $25\text{ }^{\circ}\text{C}$).

Equipment

For electron tomography a FEI Tecnai 20 electron microscope was used with a LaB₆ tip at an accelerating voltage of 200 kV in bright field mode. Transmission electron microscopy (TEM) was performed on a FEI Tecnai 12 with a tungsten tip, operating at 120 kV . For scanning transmission electron microscopy (STEM) a FEI Tecnai 20F with a Field Emission Gun (FEG) was used, operating at 200 kV in secondary electron mode (SE-STEM) or in high-angle annular dark-field mode (HAADF-STEM). Confocal analysis was performed on a Leica SP2 confocal microscope with an $100\times$ ($\text{NA} = 1.4$) oil immersion Leica confocal objective. For centrifugation of nanoparticles a Hettich zentrifugen Rotina 46 S @ 4000 RPM was used, while for centrifugation of nanoparticle clusters an Eppendorf centrifuge 5415c was used @ 1000 RPM .

Synthesis of cobalt iron oxide and core-shell semiconductor nanoparticles

6.0 nm cobalt iron oxide nanoparticles with an oleic acid capping were synthesized according to the method described by Bodnarchuk *et al.* (1), using hexadecene as solvent instead of octadecene, and washed three times using ethanol/cyclohexane (1:1 v/v) by sedimenting and redispersing using centrifugation. Electron microscopy analysis showed nanoparticles with a 6.0 nm core with a standard deviation of 0.3 nm resulting in a polydispersity of 4.8% . The effective diameter was found to be 9.0 nm as a result of the oleic acid ligand molecules (measured over multiple particles in close packed structures). This resulted in an effective polydispersity of 3.2% (see Supplementary Fig. 1 left).

For experiments on core-shell semiconductor nanoparticles, 12.4 nm CdSe@ZnS

multishell nanoparticles with a polydispersity of 8.1 % were used, synthesized according to Xie *et al.* (2). These particles were capped with oleic acid resulting in an effective diameter of 14.5 nm and an effective polydispersity of 6.9 % (see Supplementary Fig. 1 right).

Synthesis of silica colloids

Fluorescent core-shell particles were made using the dye fluorescein isothiocyanate (FITC) according to the method by van Blaaderen *et al.* (3). The cores had a diameter of 434 nm and were used in seeded growth to arrive at a final diameter of 1.32 μm . Successive silica shells were grown on the cores according to the Giesche method (4). Electron microscopy analysis showed silica colloids with a diameter of 1.32 μm with a polydispersity of 1.7 % (see Supplementary Fig. 2).

Experimental self-assembly in spherical confinement

For a typical nanoparticle self-assembly experiment 5 mg of 6.0 nm cobalt iron oxide nanoparticles were dispersed in 1 ml of cyclohexane and added to a mixture of 800 mg of dextran and 120 mg of sodium dodecyl sulfate in 20 ml of de-ionized water. The resulting emulsion was agitated by shear with a shear rate of $\dot{\gamma} = 1.56 \times 10^5 \text{ s}^{-1}$, using a Couette rotor-stator device (gap spacing 0.1 mm) following the procedure and equipment described by Mason and Bibette (5). The emulsion was then heated to 68 °C and kept at this temperature for 4 hours to evaporate the inner cyclohexane phase. The suspension was then allowed to cool to room temperature. The resulting supraparticle suspension was washed twice, sedimenting and redispersing in de-ionized water. Examples of nanoparticle clusters are shown in Supplementary Fig. 1 and additional images of a FCC supraparticle are shown in Supplementary Fig. 3.

Self-assembly experiments were also performed with semiconductor nanoparticles yielding similar results as the cobalt iron oxide nanoparticles, as shown in Supplementary Fig. 4. For a typical self-assembly experiment 5 mg of semiconductor nanoparticles were dispersed in 1 ml of cyclohexane. Consecutively, the same steps were performed as described for the cobalt iron oxide nanoparticle self-assembly experiments. After self-assembly, bright red photoluminescent supraparticles were observed with a peak in intensity at the same wavelength ($\lambda_{\text{max}} = 650 \text{ nm}$) as for the constituent nanoparticles.

For a typical colloidal self-assembly experiment 5 mg of silica colloids were dispersed in 25 μL of de-ionized water. The suspension was then added to 150 μL of hexadecane containing 1.5 mg Span 80. The suspension was shaken slightly by hand. The inner water phase of the emulsion droplets was allowed to evaporate over 10 days with slight agitation (shaking) every 48 hours to prevent droplets

from sticking to the glass vial. The resulting suspension was washed by sedimenting and redispersing in water. Images of typical colloidal clusters are shown in Supplementary Fig. 5.

Electron microscopy sample preparation

To prepare sample for electron microscopy, 16 nm gold nanoparticles were deposited (from ethanol) on the transmission electron microscopy (TEM) grid prior to the sample deposition. These functioned as fiducial markers for tomographic reconstruction. 3 μL of the supraparticle suspension in de-ionized water were deposited on the TEM grid and plunge frozen in liquid ethane using a Vitrobot Mark2 plunge freezer. The sample was then freeze-dried over a period of 4 hours under vacuum at -90°C and subsequently allowed to warm to room temperature (RT) prior to TEM analysis.

Scanning transmission electron microscopy analysis

To study the surface order of the nanoparticle supraparticles scanning transmission electron microscopy (STEM) was used operating in secondary electron mode (SE-STEM). To study the interior of the supraparticles, images were acquired in HAADF-STEM mode. Prior to analysis the samples were treated with a plasma cleaner for 2 x 6 seconds to remove excess organic residue. For analysis a Tecnai 20F electron microscope with a FEG was used in STEM mode operating at an accelerating voltage of 200 kV.

Determining nanoparticle and colloid coordinates

The internal structures of nanoparticle assemblies were studied using electron tomography. A Tecnai 20 electron microscope was used with a LaB₆ tip, operating at an accelerating voltage of 200 kV. Tilt series were acquired over 140° using FEI Explore 3D software. The reconstruction was made using IMOD, a software package available via the Boulder Laboratory, Colorado (6, 7). For reconstruction a Simulative Iterative Reconstructive Technique (SIRT) was used with 20 iteration steps.

The orthoslices, from the tomographic reconstruction, were filtered for noise reduction. Using ImageJ, the following filter operations were used: Invert contrast, Subtract background (50 px), a Bandpass filter (30 px high, 3 px low), a Gaussian blur (3 px) and Enhance contrast.

The internal structure of supraparticles consisting of silica colloids was studied using confocal microscopy. For this the supraparticle samples were suspended in an index matching solution of 40/60 hexadecane/toluene inside a capillary. 3D

z -stacks of the colloidal samples were acquired using a Leica SP2 confocal microscope, with a 100x (NA = 1.4) oil objective (see Supplementary Fig. 5).

The tracking software used to find particle positions in both the colloidal and nanoparticle assemblies, was based on the tracking algorithm of Crocker and Grier (8), but extended to 3D as schematically described in (9) (see Supplementary Fig. 6).

In Supplementary Fig. 3 (of the main text), for the surface of the nanoparticles assembly (bottom row, second from the left) we found the particle positions from the SE-STEM image using the 2D tracking shown in Supplementary Fig. 7.

Identifying crystalline domains

We use a local bond orientational order parameter based on spherical harmonics $Y_{l,m}$ to identify the solid-like particles, i.e. particles with a solid-like environment, as introduced in (10). We set $l = 6$, as the face-centered-cubic (FCC) crystal structure is the stable solid phase. Subsequently we use a cluster algorithm (11) to discriminate different crystalline domains.

For each particle i , we calculate

$$q_{6m}(i) = \frac{1}{N_b^i} \sum_{j=1}^{N_b^i} Y_{6,m}(\mathbf{r}_{ij}),$$

where N_b^i is the number of neighbors of particle i , i.e. all particles j with a center-of-mass distance $|r_{ij}| < 1.4\sigma$ with respect to particle i , and σ is the particle diameter. The correlation between two particles i and j is quantified by:

$$d_6(i, j) = \frac{\sum_{m=-6}^{+6} q_{6m}(i) q_{6m}^*(j)}{\left(\sum_{m=-6}^{+6} |q_{6m}(i)|^2 \right)^{1/2} \left(\sum_{m=-6}^{+6} |q_{6m}(j)|^2 \right)^{1/2}},$$

and for each particle the number of connections is evaluated via

$$n_{con}(i) = \sum_{j=1}^{N_b^i} \mathcal{H}(d_6(i, j) - d_c),$$

where $\mathcal{H}$ is the (Heaviside) step function, and where we fix the correlation threshold $d_c = 0.6$. A particle i is defined as solid-like if $n_{con}(i) \geq 7$.

Different crystal domains are then identified by using an additional constraint to

our criterion, reflecting the fact that the local environment of two neighbouring solid-like particles should display more similarity if they belong to the same crystalline domain than if they are part of different ones. According to this criterion we add particle j (already qualified as a solid-like particle) to the same crystalline domain of particle i if $d_6(i, j) \geq 0.9$.

Molecular dynamics simulations

We perform event-driven molecular dynamics (EDMD) simulations (12) of hard spheres with a diameter σ confined in a spherical cavity. To model the spherical confinement, we use two types of external potentials that act on the hard spheres. In the first case, we treat the confinement as an impenetrable hard spherical wall of radius R . In the second case, we used a soft repulsive wall-particle potential with a functional form $1/|R - r|^n$ with $n \geq 2$, and an interaction range $\leq 2\sigma$, with r the center-of-mass distance of the particle with respect to the center of the cavity. The soft repulsive wall-particle interaction should describe more accurately the deformable emulsion droplet interface compared to the purely hard spherical wall. To mimic the solvent evaporation of the emulsion droplets, the radius of the spherical cavity is slowly reduced at a constant compression speed $v \leq 10^{-4} \sigma/\tau$, where $\tau = \sqrt{m\sigma^2/k_B T}$ is the EDMD time unit, m the mass of a particle, k_B the Boltzmann constant and T the temperature. We use an Andersen thermostat to keep the temperature of the system constant during the simulations.

In addition, we also perform EDMD simulations at fixed density, in which both the number of particles N and the cavity volume V are kept fixed, in order to investigate possible kinetic effects due to the compression. Our results do not alter upon changing the compression speed of the spherical confinement, provided the compression rate is sufficiently slow that the system remains in quasi-equilibrium during the self-assembly process.

Our simulations show clearly that icosahedral ordering arises spontaneously for hard spheres in both a hard and a soft repulsive spherical confinement as shown in Supplementary Fig. 10. Additionally, we clearly observe the structural transition from icosahedral to FCC clusters. We note that the clusters obtained in a soft repulsive spherical confinement have a higher fraction of crystalline particles and a more pronounced icosahedral symmetry than the clusters confined in a hard cavity as the geometrical confinement is less severe for the particles close to the surface, i.e., the clusters become more faceted. Similarly, the anti-Mackay surface termination is also more evident in the case of a soft confinement.

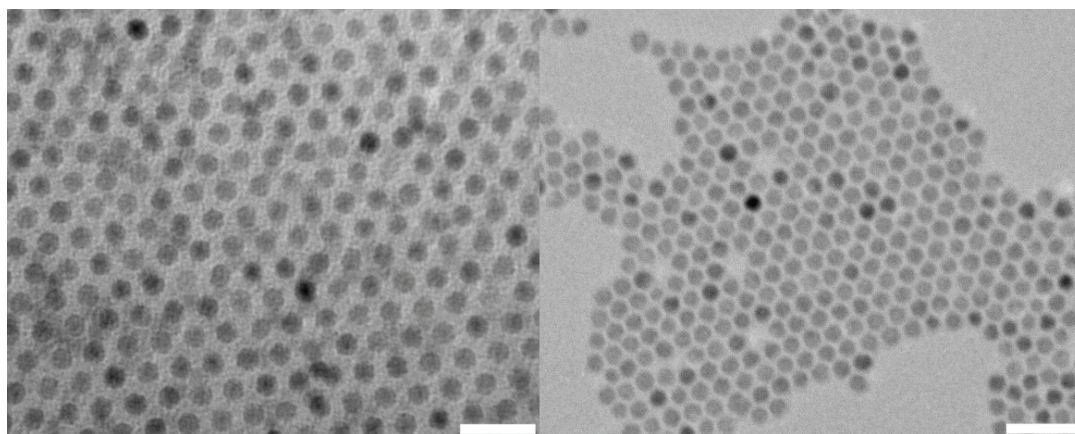
Moreover, we perform simulations with a short-ranged (1σ) attractive potential between interface and particles (maximum depth $10k_B T$). As shown in Supplementary Fig. 11, we can conclude that the type of interface (repulsive or attractive) does not play a vital role in the stabilization of icosahedral symmetry.

Finally, we study the crystallization process using our EDMD simulations. We determine the fraction of crystalline particles in the surface layer near the confining wall, the first three sublayers from the surface, and the inner part of the cavity as a function of packing fraction. In Supplementary Fig. 12, we show results for the formation of a Mackay icosahedral cluster of 2000 hard spheres in a hard spherical confinement. We find that the melting and refreezing of the surface layers is less pronounced than for the anti-Mackay icosahedral cluster as shown in Fig. 4.

Free energy calculations

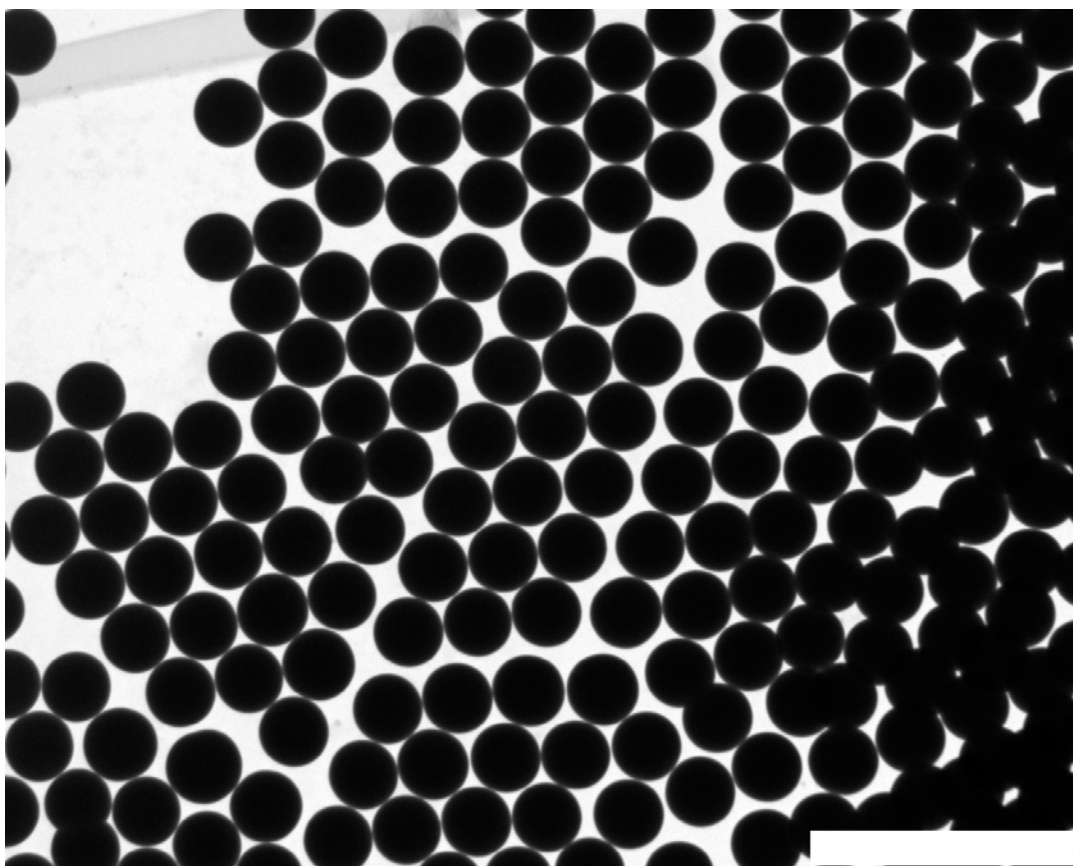
To explore whether the structures we found were equilibrium structures, we calculate the free energy of both FCC clusters as well as icosahedral clusters in a hard spherical cavity using a 3-step thermodynamic integration method proposed by Schilling and Schmid (13). In particular, this method allows us to calculate the free-energy difference between our system, e.g. spherically confined hard spheres, and a reference system for which the free energy is known exactly. The reference system we use consists of non-interacting particles attached, via a linear well potential, to lattice sites. The lattice sites are taken from representative configurations of the system featuring the desired cluster symmetry. In the first integration step, the coupling between the particles and their respective lattice sites is slowly turned on. To accurately perform this integration, we sampled more than 300 values of the coupling constant, and fit the results with an Akima spline before performing the integration. The other two integration steps consist of slowly switching off the hard interactions in the model, i.e. the particle-particle and particle-wall interactions. To do this, we followed the method described in (14). These last two integrations were done using a standard 20-points Gauss-Legendre integration scheme. The results of our free-energy calculations are shown in Supplementary Table 1. They clearly show that for systems with 1500 - 4000 particles, clusters with icosahedral symmetry have lower free energy than the FCC clusters, i.e. are more stable, for the packing fractions we studied. To get an estimate of the error in our free energies, we repeated our calculations on five independent realizations of each cluster geometry.

Supplementary Figures



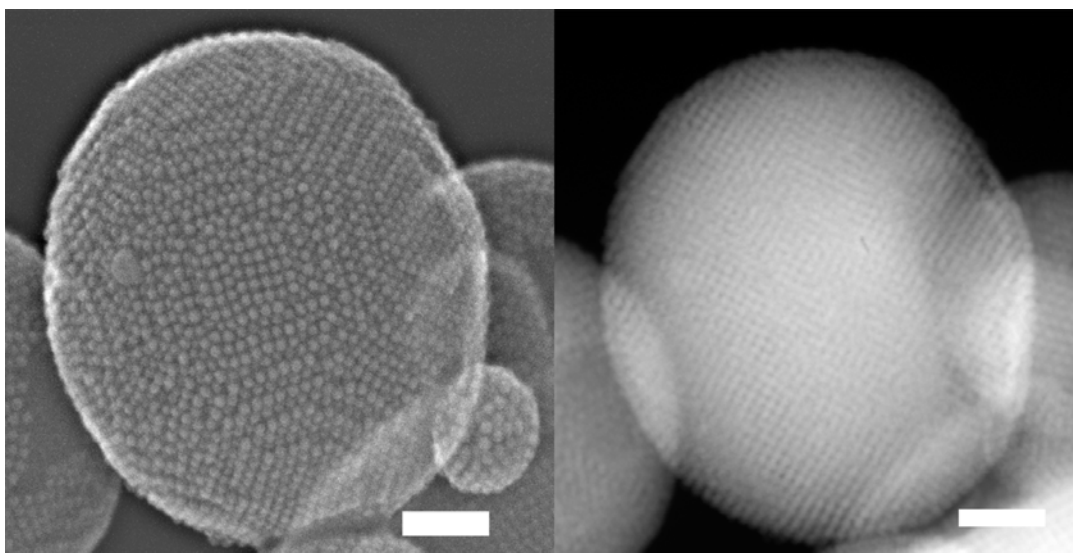
Supplementary Figure 1: Electron micrograph of nanoparticles

Left: Electron micrograph of 6.0 nm cobalt iron oxide nanoparticles with an effective polydispersity of 3.2 % (scalebar = 20 nm). **Right:** Electron micrograph of 12.4 nm CdSe@ZnS semiconductor nanoparticles with an effective polydispersity of 6.9 % (scalebar = 50 nm).



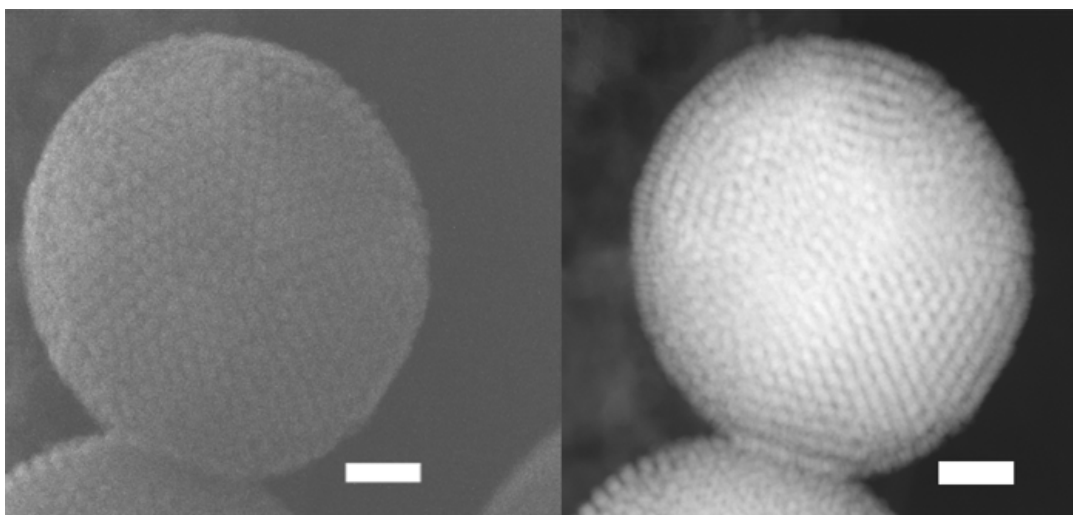
Supplementary Figure 2: Electron micrograph of silica colloids

Electron micrograph of $1.32\ \mu\text{m}$ silica colloids with a polydispersity of 1.7 % (scale-bar = $5\ \mu\text{m}$).



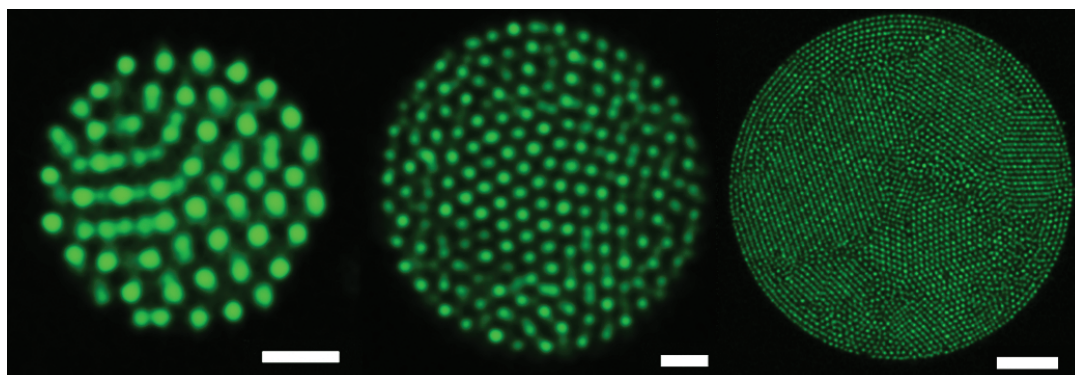
Supplementary Figure 3: Electron micrographs of an FCC supraparticle of cobalt iron oxide nanoparticles

Left: Secondary electron scanning transmission electron microscopy (SE-STEM) image showing a highly ordered surface with hexagonal (111) and square (100) packed domains. The typical step edges are observed. **Right:** HAADF-STEM image showing the highly ordered internal structure of the assemblies with crystallinity throughout.



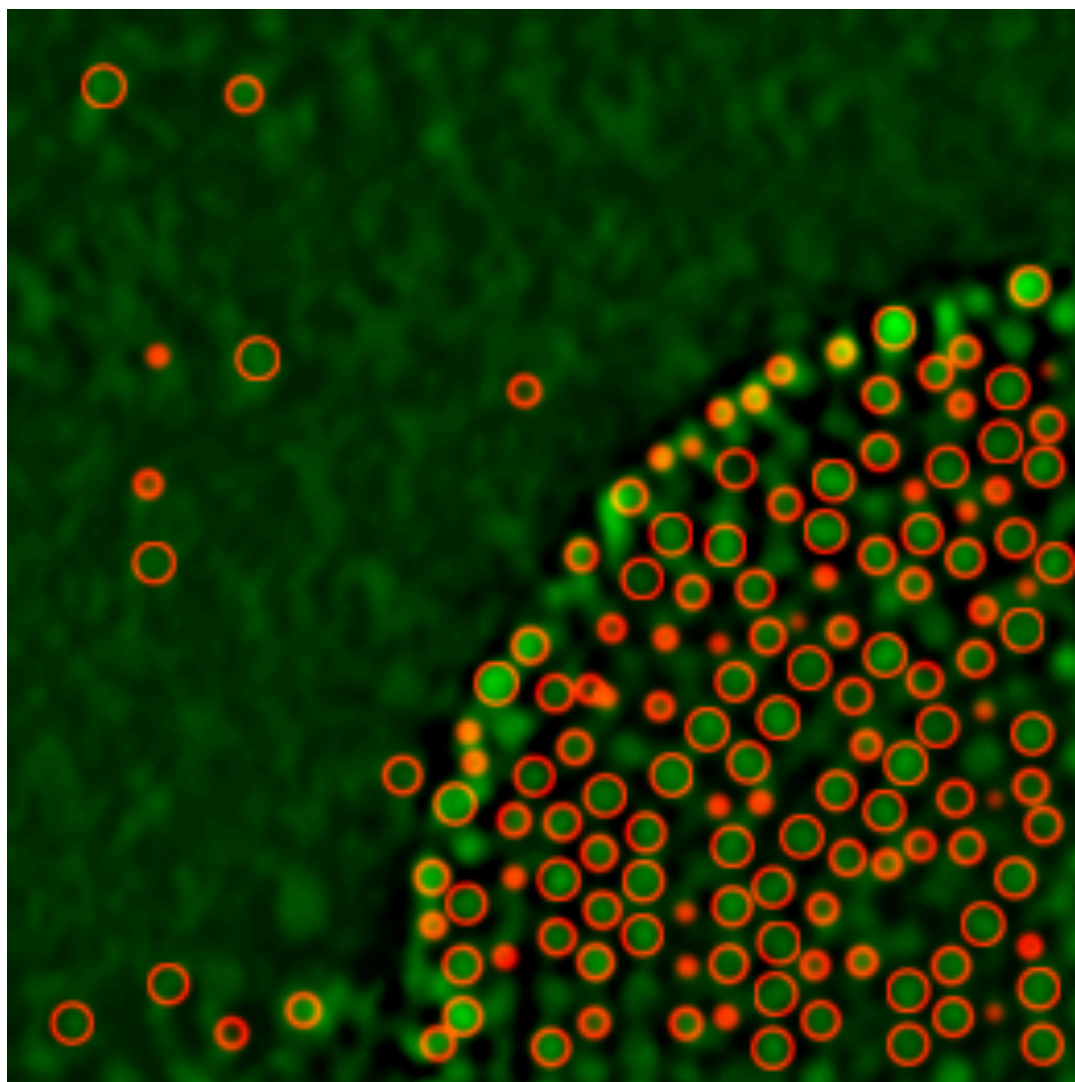
Supplementary Figure 4: Electron micrographs of an icosahedral supraparticle of semiconductor nanoparticles

Similar packing behaviour was observed when CdSe@ZnS core-shell semiconductor nanoparticles were used. **Left:** SE-STEM image of semiconductor nanoparticles self-assembled into a supraparticle. **Right:** HAADF-STEM image of the same supraparticle showing the characteristic wedge shaped domains pointing to the center of the cluster.

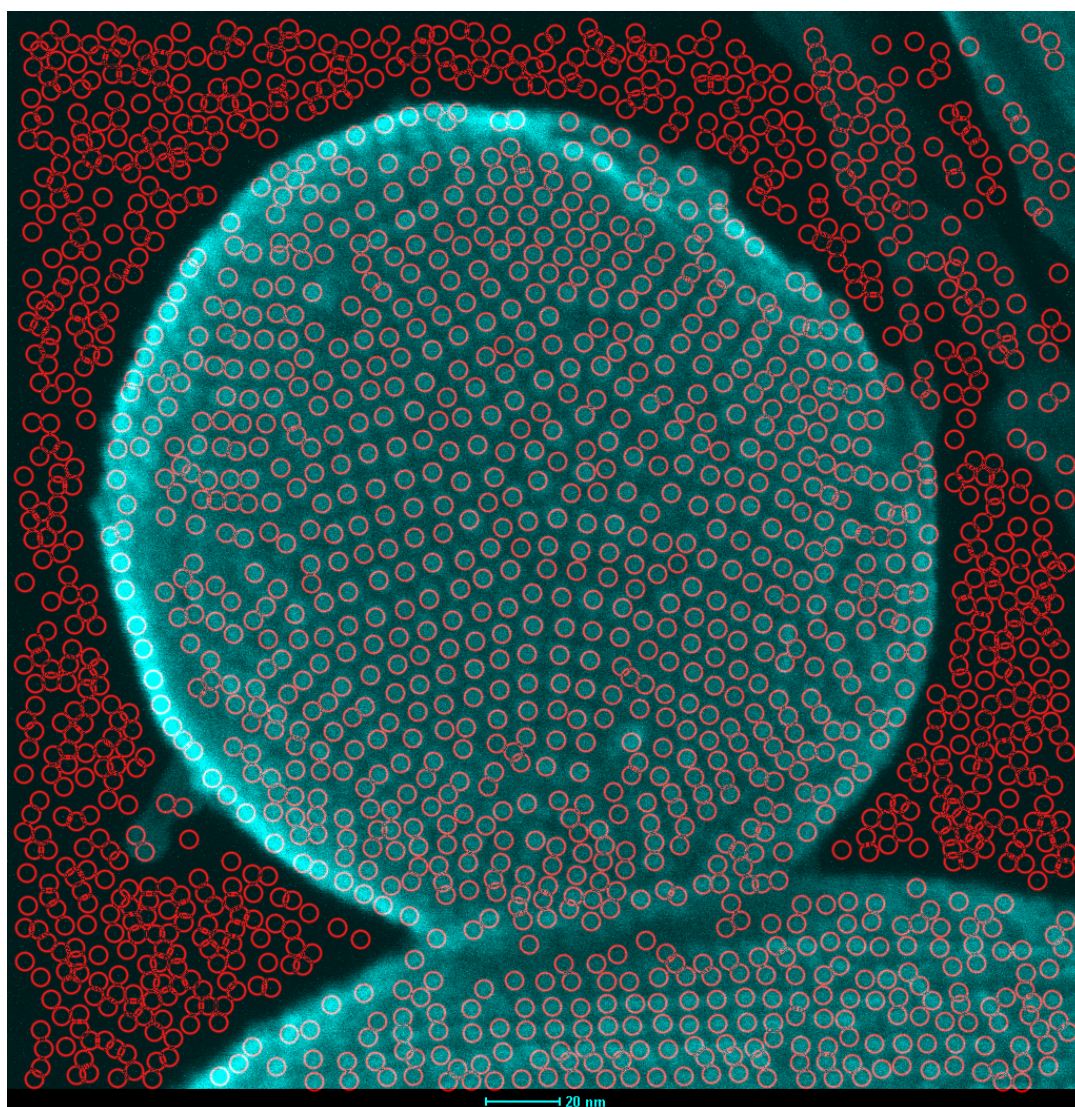


Supplementary Figure 5: Confocal images of silica colloids self-assembled into supraparticles

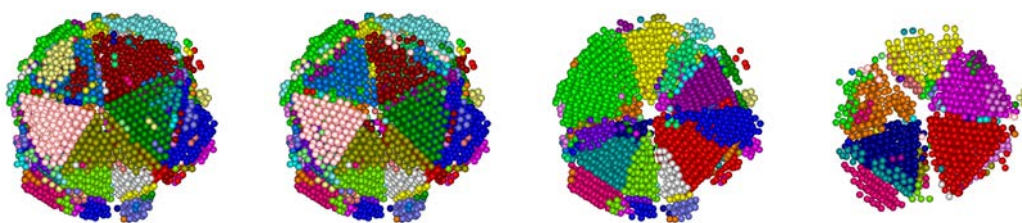
A plane through each assembly is shown. The different crystalline symmetry is clearly observed. **Left:** (Mackay) Icosahedral symmetry (scalebar = $5\ \mu\text{m}$). **Center:** Icosahedral symmetry with anti-Mackay surface termination (scalebar = $5\ \mu\text{m}$). **Right:** Large FCC crystalline domains (scalebar = $25\ \mu\text{m}$). Images were Gaussian-blurred and brightness was increased to better visualize colloids.

**Supplementary Figure 6: Nanoparticle tracking**

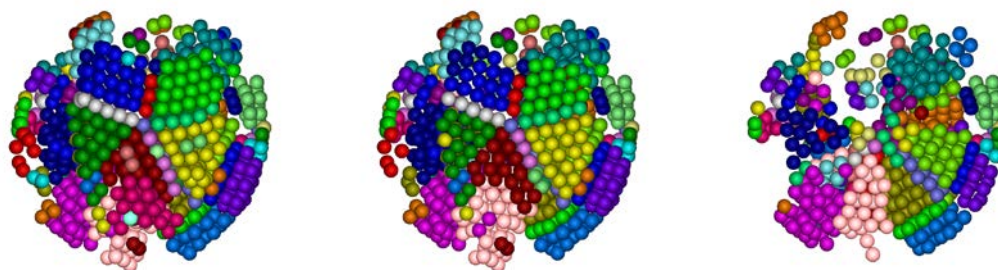
Determining coordinates of cobalt iron oxide nanoparticles. Particles with a red circle are recognized and their positions are determined in 3D. Those without a circle are either above/below the plane or are not recognized by the tracking software. Typical difficulties in particle tracking arose from the noise generated by the tomographic reconstruction process or due to the small gap between the particles.

**Supplementary Figure 7: Surface tracking from SE-STEM image**

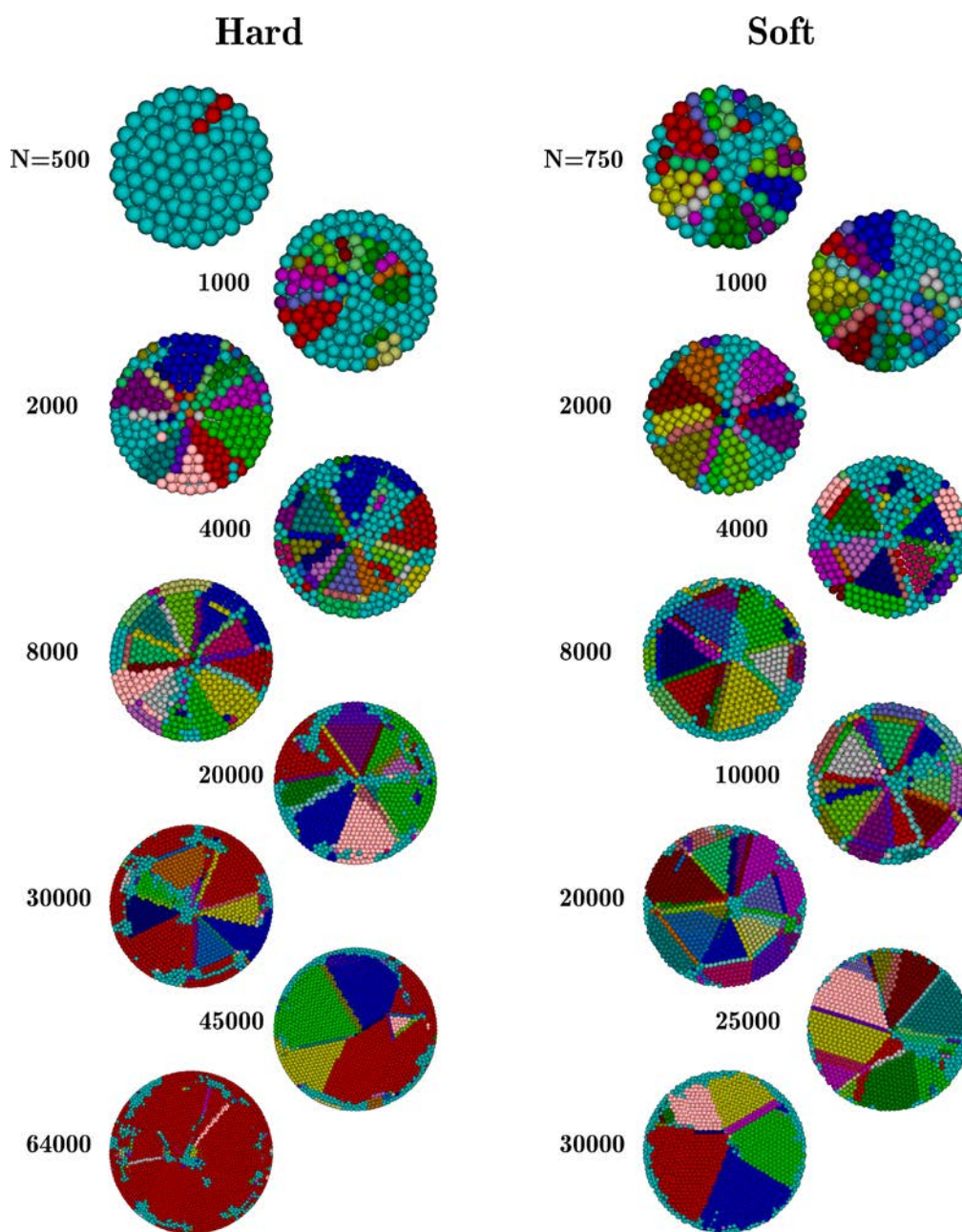
2D tracking of the cobalt iron oxide nanoparticles from the SE-STEM image shown in Fig. 1 (central panel). This method was used to obtain in a more accurate way the coordinates used for one of the images of Fig. 3 (bottom row, central panel).

**Supplementary Figure 8: Cross-sections of a nanoparticle cluster**

Cuts through an icosahedral cluster of core-shell semiconductor nanoparticles. To make these figures, we obtained the 3D coordinates of the individual particles from electron micrographs using particle tracking software (8, 9). Different colors indicate different crystalline domains, as determined via a bond-orientational order parameter (10). Particles identified as non-crystalline were hidden. The first image shows the outside of the cluster, while the next three show progressively deeper cuts through the cluster, where 5, 10, and 5 tetrahedra are intersected, respectively. Note that the five-fold symmetry persists throughout the entire structure.

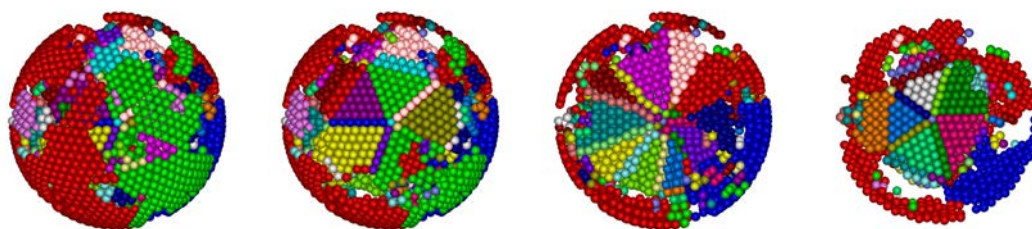
**Supplementary Figure 9: Cross-sections of a colloid cluster**

Cuts through an icosahedral cluster of core-shell silica colloids. To make these figures, we obtained the 3D coordinates of the individual particles from confocal microscopy images using particle tracking software (8, 9). Different colors indicate different crystalline domains, as determined via a bond-orientational order parameter (10). Particles identified as non-crystalline were hidden. The first image shows the outside of the cluster, while the other two images show cuts through the cluster, where 5 and 10 tetrahedra are intersected, respectively. Note that deeper in, the cluster is largely non-crystalline due to sedimentation effects, as mentioned in the main paper.



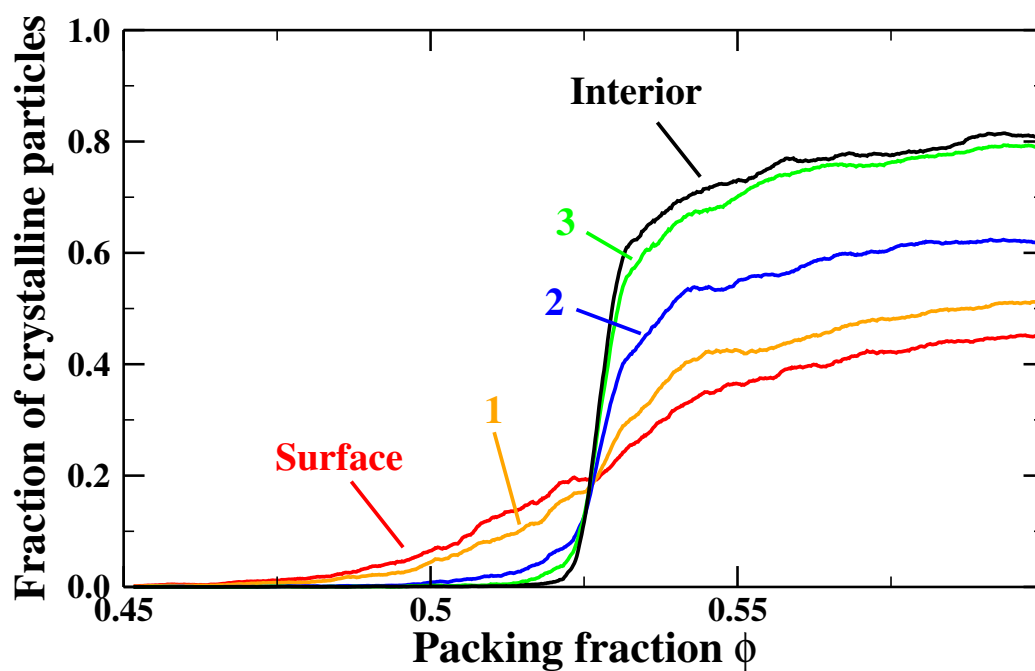
Supplementary Figure 10: Clusters from EDMD simulations

A cut through the middle of the cluster is shown for different system sizes (N is the total number of particles). **Left:** A hard spherical confinement. **Right:** A soft repulsive spherical confinement.



Supplementary Figure 11: Clusters from EDMD simulations with an attractive wall

Snapshots of a cluster of $N = 8000$ particles in a system where the wall-particle interaction was modeled by a short-ranged attractive potential. The image on the left shows the outside of the cluster, while the next three show progressively deeper cuts through the cluster, where 5, 10, and 5 tetrahedra are intersected, respectively. Particles identified as non-crystalline are hidden. Similar results were observed in smaller clusters ($N = 4000$).



Supplementary Figure 12: Crystallization process of Mackay icosahedral clusters

Crystallization process studied by event-driven molecular dynamics simulations of 2000 hard spheres in a hard spherical confinement that slowly shrinks in time. Fraction of crystalline particles in the surface layer, the first three sub-layers and the interior as a function of the packing fraction ϕ .

Supplementary Table

N	ϕ	$\Delta F = F_{\text{FCC}} - F_{\text{ico}} [k_B T/N]$
1500	0.551	0.026 ± 0.020
1500	0.571	0.036 ± 0.012
3000	0.554	0.036 ± 0.006
4000	0.537	0.030 ± 0.006
4000	0.558	0.038 ± 0.009
4000	0.572	0.034 ± 0.007

Supplementary Table 1: Free-energy differences

Free-energy difference between FCC-like and icosahedral-like arrangements of N hard spheres confined in a hard spherical cavity. Clusters with a global icosahedral symmetry are thermodynamically more stable than FCC clusters for the cluster sizes and packing fractions that are studied here.

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