On Moduli Stabilization in Type II B
String Theory
(Master Thesis)

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Abstract

Different scenarios for stabilization of moduli fields in the context of type II B Calabi-Yau flux compactifications are discussed in detail. Most attention is drawn to the stabilization of Kähler moduli in particular. In this context we first discuss the initial KKLT proposal [20] for stabilization of 4-cycle volumes and afterwards we turn to the Large Volume Scenario [21, 22]. The different possibilities of obtaining Anti de Sitter minima for very big volumes of the internal manifold are shown numerically. Next, we consider the case of a Calabi-Yau orientifold that allows for many axionic moduli present. It is shown that these can also be stabilized by the standard techniques when we include $\alpha'$-corrections to the Kähler potential $K$ and nonperturbative D3-instanton contributions to the superpotential $W$. At last, we comment on the possible influence of worldsheet instantons on the process of moduli stabilization.
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Introduction

One of the main problems of string theory from its birth to present day has been the lack of testable predictions. Although it is a consistent quantum theory of gravity, it contains many features that seem bizarre and unrealistic such as the requirement of 10 space-time dimensions, the extended objects like D-branes and so on. In particular, string theory strikes with its great generality, i.e. it is not only supposed to describe our universe but also a large set of new possibilities that clearly lead to different physical laws and could only be realized in different ”universes” or non-causally connected parts of our own universe. This is the famous string theory landscape, which we will describe in more detail in this work. At this point the critics of string theory can rightly claim that it is in fact a ”Theory of Anything” rather than the so proclaimed Theory of Everything. In order to prove the critics wrong, one would have to first find unambiguously the cases in which the conditions of our universe are reproduced within string theory and further test experimentally all new predictions. To ”reproduce the conditions” in our universe, put in more technical words, means that one has to find a solution of string theory with 4 large (and 6 small) space-time dimensions and small positive cosmological constant (corresponding to slowly accelerating universe), in which the Standard Model holds. This just means that all matter and forces should be the ones that we ”see” around us.

It unfortunately turns out that this task is not that easy to reach and physicists and mathematicians have been working on it for more than 20 years now. The first step of finding string theory solutions with 4 large and 6 small and invisible dimensions has been well-understood by now. It follows the ideas of Kaluza-Klein theory [1, 2], developed in the beginning of the 20th century. One uses an ansatz for the size and shape of the small dimensions and looks how different the resulting 4-dimensional picture is. The usual ansatz used in string theory (for reasons that will be explained later) is that the 6 dimensions form a compact 6 dimensional hypersurface, called a Calabi-Yau
Figure 1.1: Diagram of the relevant fields contributing to the topic of Moduli Stabilization.
manifold [3]. The second step of finding a tiny positive cosmological constant leads us to the main topic of this thesis, namely the process of moduli stabilization. This is still an open subject and in the following we will try to explore it and show that the accomplishment of positive cosmological constant is indeed possible. The main problem is the large number of scalar massless fields that arise from the compactification to 4 dimensions. These cannot be accommodated in the Standard Model and one needs to find a way to generate a mass for them, sufficiently high so that we cannot observe them experimentally (i.e. at least $114\, GeV$). The third step of reproducing the Standard Model is already beyond the scope of the present paper and can only be done properly after stabilizing all the scalar fields. However, there has been some considerable progress in this direction also, and we will mention it briefly in the end of this section.

The field of moduli stabilization requires certain preknowledge of several different aspects of string theory and topology on smooth manifolds, as shown schematically on Fig. (1.1). We will try to explain in some detail all ingredients that will be used in later sections, but nevertheless the reader is expected to possess some knowledge in string theory, quantum physics and differential geometry before trying to understand this paper. An introductory course in string theory should be sufficient. For a comprehensive introductory review on the subject one can look at [4] for example.

1.1 Low-energy effective actions

There are 5 basic types of string theories related with each other by various duali-
ties: type II A, type II B, type I, heterotic $E_8 \times E_8$ and heterotic $SO(32) \times SO(32)$ strings. The type II A and the heterotic $E_8 \times E_8$ theory at strong coupling approach a common 11-dimensional limit, called M-theory [5]. In analogy, type II B exhibits 12-dimensional underlying theory at strong coupling, in turn called F-theory [6]. M-theory and F-theory do not contain any strings as dynamical objects so they cannot be called string theories. Unfortunately not much more is known about them and their understanding remains an open question (for recent development on F-theory cf. [7]). All five string theories give rise to a certain field spectrum in flat 10-dimensional Minkowski space coming from the infinite tower of superstring states. In the low-
energy limit when we take the $\alpha' \to 0$ the masses of all massive string states become very large and one can safely neglect them as they are too heavy to occur. Therefore it is a good approximation to consider only the interactions between massless modes as the complete theory. These result in different massless supersymmetric theories cor-
responding to the different string theory types. Since all of these contain the graviton in their massless spectrum, we obtain in this way supergravity theories, or so called low-energy effective actions. Although the theories are not fundamental, one can understand important features of the underlying fundamental string theories, e.g. some of the string dualities can be easily illustrated via the effective actions. When working with low-energy effective actions we have to always be consistent and make sure we remain in weak coupling at all times. It further turns out that one can formulate the low-energy effective action of M-theory, while such does not exist in the F-theory case. Here we should stress that in fact the resulting supergravity actions are generically non-renormalizable by power-counting as one could expect since otherwise quantizing gravity would be a far easier task\(^1\). However, for such kind of effective non-fundamental theories non-renormalizability is not a problem. This just means that there are infinite perturbative quantum corrections coming from higher order Feynman loop-diagrams that can be ignored if we stick to low-energies. We will however see that the low-order perturbative corrections and non-perturbative semiclassical contributions (instantons) will be important. Unfortunately, there is no prescription where exactly one has to start considering the quantum corrections and to which order. The best strategy used in literature is to stick to the tree-level results from the supergravity action and only consider quantum corrections where they play crucial importance.

Let us now concentrate on deriving one specific low-energy effective action - the one coming from type II B theory. In the next sections we will exclusively discuss this case, so here we will try to describe the action in some detail. Note that many features of the derivation are generic for the other string theories, so one sees the general process of finding all effective actions. The leading principles to construct this theory come from supersymmetry and gauge invariance. The particle content of the type II B supergravity is derived from the massless spectrum of the corresponding superstring type. The fermionic part consists of two left-handed Majorana-Weyl gravitinos and two right-handed Majorana-Weyl dilatinos. As supersymmetry holds and all fermionic degrees of freedom correspond exactly to bosonic ones, specifying either part of the effective actions completely determines the other one. In this case there are 32 supersymmetry generators, i.e. we are in the case of \( \mathcal{N} = 2 \) supergravity in 10 dimensions. We will then concentrate on the bosonic part from here on, keeping in mind the fermionic counterparts. In the bosonic spectrum we have NS-NS and R-R bosons. The NS-NS bosons are the metric \( g_{MN} \), a two-form \( B_2 \) (with corresponding field strength \( H_3 = dB_2 \)) and a dilaton \( \Phi \). The R-R sector consists of corresponding form fields \( C_0, C_2, \) and \( C_4 \), the latter having a self-dual field strength \( F_5 \). The last remark actually introduces an

\[^1\]A notable exception seems to be maximally supersymmetric (\( \mathcal{N} = 8 \)) supergravity in 4 dimensions [8].
unfortunate complication, since an action of the form
\[ \int |F_5|^2 d^{10}x = \int F_5 \wedge *F_5, \] (1.1)
cannot incorporate the self-duality and thus describes twice as many degrees of freedom as desired. To overcome this problem one can proceed in several ways. It is possible to drop the idea of constructing an action and only find the field equations and supersymmetry transformations. This is perfectly valid to do since we are only constructing an effective theory not to be inserted in a path integral. We will however go for the second option - present an action that leads to the correct equations of motion only after imposing the self-duality as an extra constraint on the system. The drawback is that the action is not supersymmetric as it stand without the constraint because of the additional bosonic degrees of freedom.

The way people have discovered this action is by first constructing the supersymmetric equations of motion. Afterwards one can find the action that reproduces those equations after imposing the self-duality constraint. This way, the bosonic part of the action is found to be [4]:
\[ S = S_{NS} + S_R + S_{CS}, \] (1.2)
where
\[ S_{NS} = \frac{2\pi}{(2\pi \sqrt{\alpha'})^8} \int d^{10}x \sqrt{-g} e^{-2\Phi} \left( R + 4 \partial_{\mu} \Phi \partial^{\mu} \Phi - \frac{1}{2} |H_3|^2 \right), \] (1.3)
\[ S_R = -\frac{2\pi}{(2\pi \sqrt{\alpha'})^8} \int d^{10}x \sqrt{-g} \left( |F_1|^2 + |\tilde{F}_3|^2 + \frac{1}{2} |\tilde{F}_5|^2 \right), \] (1.4)
\[ S_{CS} = -\frac{2\pi}{(2\pi \sqrt{\alpha'})^8} \int C_4 \wedge H_3 \wedge F_3. \] (1.5)

We used the conventions \( H_3 = dB_2, F_{n+1} = dC_n \) and
\[ \tilde{F}_3 = F_3 - C_0 H_3, \]
\[ \tilde{F}_5 = F_5 - \frac{1}{2} C_2 \wedge H_3 + \frac{1}{2} B_2 \wedge F_3. \] (1.6)

With these definitions the self-duality constraint that (1.2) is subject of is
\[ \tilde{F}_5 = *\tilde{F}_5. \] (1.7)

### 1.2 Calabi-Yau manifolds

A Calabi-Yau \( n \)-fold is defined as a Kähler manifold with \( n \) complex dimensions (\( 2n \) real dimensions) and vanishing first Chern class \( c_1 = 0 \), i.e. it is a special type of Kähler manifold. In turn a Kähler manifold is a complex manifold with a Riemannian
(positive-definite) metric and a symplectic form (a closed, nondegenerate 2-form) on the underlying real manifold. It is important condition that these 3 structures (the complex structure, the metric, and the symplectic form) are mutually compatible. Using this definition and the constraint that \( c_1 = 0 \), Calabi and Yau conjectured and proved respectively that a compact manifold of this kind admits a Kähler metric of \( SU(n) \) holonomy. Such a manifold is necessarily Ricci flat, which will be discussed in more detail later. The metric of any Kähler manifold is special because it can be encoded in a scalar function, called the Kähler potential \( K \). There always exist special complex coordinates \( z^\alpha, \bar{z}^{\bar{\alpha}} \) for which the metric is given by:

\[
g_{\alpha \bar{\beta}} = \partial_\alpha \bar{\partial}_{\bar{\beta}} K. \tag{1.8}\]

We will also need the fact that a Kähler manifold is of Calabi-Yau type if and only if it admits a nowhere vanishing holomorphic \( n \)-form \( \Omega \). Locally,

\[
\Omega(z^1, z^2, ..., z^n) = f(z^1, z^2, ..., z^n) dz^1 \wedge dz^2 \ldots \wedge dz^n. \tag{1.9}
\]

A central role here will be played by the elements of the de Rham cohomology of the Calabi-Yau’s, i.e. the harmonic \( k \)-forms on the manifolds. The reason lies in the process of Kaluza-Klein compactifications and will be explained in the next subsection. For this reason we first need to develop some language and mathematical apparatus. The fundamental topological numbers specifying the dimension of each cohomology are the Betti numbers \( b_k \). For a manifold \( M \) with a metric, \( b_k \) counts the number of the linearly independent harmonic \( k \)-forms on \( M \), i.e. \( \text{dim}(H^k(M)) = b_k \). When \( M \) is a Kähler manifold, the Betti numbers can be decomposed in terms of Hodge numbers counting also the holomorphicity of the respective \( k \)-forms:

\[
h^{p,q} = \text{number of harmonic } (p, q)\text{-forms}. \tag{1.10}\]

Clearly,

\[
b_k = \sum_{p=0}^k h^{p,k-p}. \tag{1.10}\]

A Calabi-Yau manifold is characterized by its set of Hodge numbers, although there are cases when inequivalent Calabi-Yau’s have the same Hodge numbers. The set of numbers itself is constrained by various cohomological identities. From the fact that the spaces \( H^p(M) \) and \( H^{n-p}(M) \) are isomorphic (by Hodge duality) it follows that

\[
h^{p,0} = h^{n-p,0}. \tag{1.11}\]

Complex conjugation of harmonic forms leads to

\[
h^{p,q} = h^{q,p}, \tag{1.12}\]

and Poincaré duality gives

\[
h^{p,q} = h^{n-q,n-p}. \tag{1.13}\]
Further we know that every compact connected Calabi-Yau has $h^{0,0} = 1$ corresponding to the constant functions and if it is simply connected (as the ones that we will consider) it has a vanishing fundamental group, leading to vanishing first homology group:

$$h^{1,0} = h^{0,1} = 0.$$  \hspace{1cm} (1.14)

With these identities the complete cohomological description is quite restricted, in the case of $n = 3$ (6 real dimensions) that we are interested in one is only left to specify $h^{1,1}$ and $h^{2,1}$. Note that $h^{1,1} \geq 1$ since all Calabi-Yau have the symplectic Kähler form $J$.

The full set of Hodge numbers is typically displayed in the form of a Hodge diamond:

\[
\begin{array}{cccccc}
 & & h^{3,3} & & & \\
 & h^{3,2} & h^{2,3} & h^{1,3} & h^{0,3} & \\
 h^{3,1} & h^{2,2} & h^{1,2} & h^{0,2} & h^{1,1} & h^{2,1} & h^{3,1} \\
 h^{2,0} & h^{1,1} & h^{0,1} & h^{1,1} & h^{2,1} & h^{3,1} & h^{2,0} \\
 h^{1,0} & h^{0,1} & h^{1,1} & h^{2,1} & h^{3,1} & h^{2,0} & h^{1,0} \\
 h^{0,0} & h^{1,1} & h^{2,1} & h^{3,1} & h^{2,0} & h^{1,0} & h^{0,1} \\
 & & h^{3,3} & & & \\
\end{array}
\]

Using the same identities the Euler characteristic of a generic compact Calabi-Yau three-fold is given by

$$\chi = \sum_{p=0}^{6} (-1)^{p} b_{p} = 2 (h^{1,1} - h^{2,1}).$$  \hspace{1cm} (1.15)

The situation for a general $n$-fold will be analogous, but as one can imagine the higher in dimension we go, the more free choices of Hodge numbers there will be.

The problem of classifying all Calabi-Yau manifolds is unfortunately still an open problem. The only examples of one-folds are the complex plane $\mathbb{C}$ and the compact two-torus $T^2$. In 4 dimensions we have the noncompact examples of $\mathbb{C}^2$, $\mathbb{C} \times T^2$ and the compact $T^4$, $K3$. In contrast to these, there are very many examples of both compact and noncompact Calabi-Yau three-folds and it is not even clear if there is finite number of them. The situation is the same for all $n > 2$. Many Calabi-Yau three-folds and four-folds have been constructed as submanifolds (hypersurfaces) of complex projective spaces $\mathbb{C}P^n$, as well as weighted complex projective spaces $\mathbb{W}C\mathbb{P}^{n}_{k_1...k_{n+1}}$. For more details of how to explicitly construct Calabi-Yau manifolds this way, as well as how to manipulate them, one can consult [9]. In Appendix A we have constructed a few simple examples this way in order to demonstrate better the procedure of subsection 2.2 (it is therefore recommended that the Appendix is read after sections 1 and 2).

For phenomenological purposes as much as for purely technical reasons one desires to break as many supersymmetries as possible while still keeping some present in the process of compactifying string theory from 10 flat dimensions to 4 flat dimensions times the 6 compact dimensions of the internal manifold. One can show that the most general
manifolds that preserve some supersymmetries are exactly the Calabi-Yau three-folds. More quantitatively, the initial ten-dimensional space time decomposes as
\[ M_{10} = M_4 \times M, \]
where \( M \) is again the compact six-dimensional internal manifold. While initially the coordinates are denoted as \( x^M \), they decompose into 4 \( x^\mu \)'s and 6 \( y^m \)'s. Assuming that \( M_4 \) is maximally symmetric, i.e. a homogeneous and isotropic four-dimensional space, it can either be Minkowski (scalar curvature \( R = 0 \)), Anti de Sitter (\( R < 0 \)), or de Sitter (\( R > 0 \)). Now, the condition for unbroken supersymmetry further imposes constraints on the space-time. Each supersymmetry charge \( Q_\alpha \) generates an infinitesimal transformation of all the fields with parameter \( \varepsilon_\alpha \). Any unbroken supersymmetry has to leave the background invariant, i.e. the vacuum state has to be annihilated by the charges. The invariance of the bosonic fields comes out trivially as these will contain at least one fermionic field that vanishes classically. So the only nontrivial condition that remains for unbroken supersymmetry is
\[ \delta_\varepsilon (\text{fermionic fields}) = 0. \]
This condition further translates into requirement for a nontrivial solution to the Killing spinor equation \( \nabla_M \varepsilon = 0 \) [4]. In short, this eventually leads to the following conditions on \( M_4 \) and \( M \). It implies that scalar curvature \( R \) should vanish in the large dimensions, i.e. \( M_4 \) needs to be Minkowski space-time. For the internal manifold we get a restriction on its metric - it needs to be Ricci-flat: \( R_{mn} = 0 \). It can be shown that the Ricci-flatness condition on the manifold (for orientable six-dimensional spin manifolds that are of interest) eventually means that \( M \) needs to have a complex structure and harmonic (1,1)-form together with a holomorphic three-form. The first two conditions ensure that \( M \) must be Kähler (the (1,1)-form is indeed the Kähler form \( J \)), while the existence of \( \Omega \) by the theorem discussed above proves that \( M \) is Calabi-Yau. Therefore, the desire of least preserved supersymmetry (\( \mathcal{N} = 2 \) in the case of type II B) translates to a space-time which is a direct product of a Minkowski space times compact Calabi-Yau manifold. Note the fact that we ruled out the possibility for dS space (\( R > 0 \)), which was our initial goal in the search of realistic model of our universe. We will later see how we can repair this by adding fluxes and breaking the supersymmetry to \( \mathcal{N} = 1 \) by orientifold projections.

### 1.3 Moduli spaces

While classifying the Calabi-Yau manifolds above, we implicitly assumed that two "different" manifolds are really two topologically different objects. Physically speaking,
one can still smoothly deform the shape and size of a manifold and this will result in the same topological object. Therefore the entire space of these topologically equivalent manifolds is referred to as a single Calabi-Yau space, even though it consists of continuously infinite number of manifolds. The parameters that control the smooth deformations of shape and size of the manifold are called moduli, and so the entire moduli space corresponds to the same Calabi-Yau manifold (space). A particularly important property of the moduli space of Calabi-Yau 3-folds is that it splits in two distinct parts - the moduli responsible for the shape of the manifold (complex structure moduli) and the ones that determine the size (Kähler-structure moduli). It will turn out that altogether these moduli parametrize the choices of expectation values of massless scalar fields in four dimensions.

Let us focus on some specific Calabi-Yau with fixed topological properties and Hodge numbers. The fluctuations around it that preserve its topological properties come from metric deformations that amount to changing the shape of the manifold (via its complex structure) and its size. To complete the string theory picture, we also have deformations of the antisymmetric tensor fields that also live on the manifold like the metric. To see the physical implications, we first need to know how Kaluza-Klein compactification works in general. Originally it used to serve for compactifying a theory on a circle from five to four space-time dimensions. This is trivially done by expanding in Fourier modes the original fields as periodic functions in the small circular dimension. Generalizing this to 6 dimensional compactification on a manifold $M$ with a scalar Laplacian $\Delta_M$, one expands the fields as sums over eigenfunctions $f^{(i)}$ of $\Delta_M$. In the simplest case for a scalar field $\phi$,

$$\phi(x, y) = \sum_i \phi^{(i)}(x) f^{(i)}(y). \tag{1.18}$$

Here, as before, $x$ refers to the four flat dimensions while $y$ are the Calabi-Yau coordinates. Initially, the ten-dimensional field $\phi$ is massless: $\Delta \phi = 0$. What previously was a ten dimensional Laplacian now splits into $\Delta = \eta^{\mu\nu} \partial_\mu \partial_\nu + \Delta_M$. Therefore the only massless four-dimensional fields that can possibly emerge from the compactification are these $\phi^{(k)}$ for which $\Delta_M f^{(k)} = 0$. The compactification of higher form fields goes in complete analogy, using higher order forms on the Minkowski space as well as the internal manifold. For a generic field $B_p$ such that $\text{mass}_{B_p} = 0$ as determined from string theory,

$$B_p(x, y) = \sum_i \left( B_p^{(i)}(x) f_0^{(i)}(y) + B_p^{(i)}(x) f_1^{(i)}(y) + \ldots + B_p^{(i)}(x) f_p^{(i)}(y) \right). \tag{1.19}$$

Furthermore, we precisely know which forms give rise to massless fields in four-dimensions, since $\Delta_M f_p = 0$ means that $f_p$ is a harmonic $p$-form. For each $p$, the number of
independent harmonic $p$-forms is exactly the Betti number $b_p$ (the dimension of the corresponding homology). We then instantly know that the massless fields in four-dimensions are only finite number of this infinite tower of Kaluza-Klein modes and the precise number is determined by the Hodge diamond of the specific Calabi-Yau manifold. For the example above, we know that for each CY manifold $b_0 = 1$, $b_1 = 0$ and $b_2 = h^{1,1}$. Therefore from the ten dimensional $B_{MN}(x, y)$ there is only one massless tensor field $B_{\mu\nu}(x)$, no massless vector fields, and $h^{1,1}$ massless scalars $B^{(i)}(x)$. These scalar fields are the moduli fields whose vacuum expectation value is not fixed and that are responsible for the deformation of the manifold. One can see from (1.19) that generically a massless $p$-form field gives rise to exactly $b_p$ moduli fields in four dimensions.

The deformations of the metric have to be considered separately since the metric is a symmetric tensor and not a two-form. However, some similarities can be found since the ten-dimensional metric gives rise to four-dimensional metric $g_{\mu\nu}$ plus a set of massless scalar fields originating from the internal components $g_{mn}$. Again no massless vector fields are generated since $b_1 = 0$. The way to analyze the fluctuations that leave the topology of the Calabi-Yau invariant is by doing a small variation of the internal metric

$$g_{mn} \rightarrow g_{mn} + \delta g_{mn},$$

and demanding that it still satisfies the Calabi-Yau condition that the metric is Ricci flat:

$$R_{mn}(g + \delta g) = 0.$$  

This equation, when written explicitly for the specific choice of a manifold (i.e. metric), leads to differential equations for $\delta g$. The number of solutions corresponds to the number of ways the metric can be deformed preserving supersymmetry and topological properties. The coefficients of these independent solutions are the moduli, the expectation values of the massless scalar fields arising from the metric. Here we can directly see how these parametrize the changes of shape and size of the Calabi-Yau manifold. The moduli fields arising from the compactification of the metric are the so-called "geometric" moduli as they have an obvious geometric interpretation, while the ones coming from the reduction of the form-fields discussed above are the "non-geometric" moduli. These two types are indistinguishable physically as they are all scalar fields which are free to take any value in space-time.

Considering more closely the deformations on Calabi-Yau 3-folds, we need to find solutions satisfying $R_{mn}(g) = 0$ together with $R_{mn}(g + \delta g) = 0$. One then eliminates the uninteresting trivial solutions of changing the coordinate system by fixing the
gauge. Using some mathematical properties of Kähler manifolds it has been shown that the equations for the mixed components $\delta g_{a\bar{b}}$ and the pure components $g_{ab}$, $g_{a\bar{b}}$ decouple (cf. [11]). Looking at them separately, we use the definition of the Kähler form $J = ig_{a\bar{b}}dz^ad\bar{z}^b$ to get a constraint on $g_{a\bar{b}}$. The volume of a Calabi-Yau 3-fold is given by

$$V_{CY} = \frac{1}{6} \int_{CY} J \wedge J \wedge J$$  

and quite naturally it must be a positive number. This should also be satisfied by the deformed $J = i(g + \delta g)_{a\bar{b}}dz^ad\bar{z}^b$. The set of these transformations satisfying $V_{CY} > 0$ is often referred to as Kähler cone in the literature, since for any admissible $J$, $cJ$ is also admissible for any positive number $c$. All five string theories contain in their spectrum the NS-NS 2-form $B$, which combines with the $J$ to give the complexified Kähler form

$$\mathcal{J} = B + iJ.$$  

(1.23)

The variations of $J$ and $B$ separately gives $h^{1,1}$ moduli fields in four dimensions as they are both two-form fields. Thus one can think of $h^{1,1}$ complex scalar fields arising in total from $\mathcal{J}$. Originally in the metric the purely holomorphic and anti-holomorphic components $g_{ab}$ and $g_{a\bar{b}}$ are zero, but one can still vary them to nonzero values thus changing the complex structure of the manifold. The complex harmonic $(2,1)$-form

$$\Omega_{abc}g_{a\bar{d}}g_{b\bar{e}}dz^a \wedge dz^b \wedge d\bar{z}^c$$  

(1.24)

can be associated to each of these variations. One can then perform the reduction to four-dimensions and find the moduli fields in the way described after Eq. (??), leading to $h^{2,1}$ complex structure moduli.

The mathematics describing the moduli spaces of Calabi-Yau manifolds is called special geometry and it has been extensively studied in literature, c.f. [9, 11]. We will not go in detail describing the special geometry, but will list the most important implications. Generally, the metric on the moduli space $G_{a\bar{b}}$ (not to be confused with the metric on the Calabi-Yau itself) is a sum of two pieces for the complex structure and Kähler deformations [10]. This means that the geometry of the moduli space has the product structure

$$\mathcal{M}(M) = \mathcal{M}^{2,1}(M) \otimes \mathcal{M}^{1,1}(M).$$  

(1.25)

The complex structure moduli space turns out to be Kähler (i.e. $G_{a\bar{b}} = \partial_a \partial_{\bar{b}}K^{2,1}$) with Kähler potential

$$K^{2,1} = -\ln \left(i \int_{CY} \Omega \wedge \bar{\Omega} \right).$$  

(1.26)
Thus we have invariance under transformations of type $\Omega \to e^{f(X)}\Omega$, since in this case $K^{2,1} \to K^{2,1} - f(X) - \tilde{f}(\tilde{X})$ and the metric is the same. Therefore the holomorphic three-form $\Omega$ is always defined up to a function $f$ that depends on the moduli space coordinates $X^I$.

The Kähler moduli space is also Kähler, but here $K^{1,1}$ is given in a bit more subtle way. First let us formally give the expansion of the complexified Kähler form in terms of the moduli fields:

$$J = B + iJ = (b^\alpha(x) + iv^\alpha(x))e_\alpha(y), \quad \alpha = 1, \ldots, h^{1,1},$$

where $e_\alpha$ is a real basis of harmonic $(1,1)$-forms. Therefore we have $h^{1,1}$ scalar moduli fields $v_\alpha$. To leading order the Kähler potential is just given by $K^{1,1} = -\ln(V_{CY})$. In order to express $V_{CY}$ in terms of our coordinates (i.e. the moduli $v_\alpha$) we need to first define the intersection numbers

$$\kappa_{\alpha\beta\gamma} = \kappa(e_\alpha, e_\beta, e_\gamma) \equiv \int_{CY} e_\alpha \wedge e_\beta \wedge e_\gamma. \quad (1.28)$$

Thus,

$$V_{CY} = \frac{1}{6} \kappa_{\alpha\beta\gamma} v^\alpha v^\beta v^\gamma. \quad (1.29)$$

However, in this case $\alpha'$ perturbative corrections to the Kähler potential are possible. These are constrained by the Peccei-Quinn symmetry [4] so that $K^{1,1}$ becomes:

$$K^{1,1} = -\ln \left( \frac{1}{6} \kappa_{\alpha\beta\gamma} v^\alpha v^\beta v^\gamma + i \frac{\zeta(3)}{2(2\pi)^3} \chi(M) \right), \quad (1.30)$$

with $\chi(M)$ the Euler number of the CY manifold. $K^{1,1}$ further receives non-perturbative corrections from instantons, but this will be discussed in more details in the next subsection.

Let us finally specify more precisely the moduli fields arising in the type II B string theory compactifications on Calabi-Yau three-folds. The resulting four-dimensional theory has $\mathcal{N} = 2$ and the resulting moduli fields arrange themselves in supermultiplets that are either vector multiplets or hypermultiplets. More explicitly, we start from the massless fields in ten dimensions:

$$G_{MN}, B_{MN}, \Phi, C, C_{MN}, C_{MNPQ}, \Psi^{(+)}_M, \bar{\Psi}^{(+)}_M, \Psi^{(-)}_M, \bar{\Psi}^{(-)}_M \quad (1.31)$$

After compactification, the remaining zero modes arrange themselves as follows.

- gravity multiplet: $G_{\mu\nu}, \Psi_\mu, \bar{\Psi}_\mu, C_{\mu\nu}$

- $h^{2,1}$ vector multiplets: $C_{\mu\nu}, G_{ij}$, fermions

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• $h^{1,1}$ hypermultiplets: $C_{\mu
u i\bar{j}}, G_{i\bar{j}}, B_{i\bar{j}}, C_{ij},$ fermions
• universal hypermultiplet: $\Phi, C, B_{\mu\nu}, C_{\mu\nu},$ fermions.

The total number of moduli fields is then $2h^{2,1} + 4(h^{1,1} + 1)$, while the massless vector fields are $h^{2,1} + 1$. The moduli space takes the form of Eq. (1.25) with $\mathcal{M}^{1,1}(M)$ a quaternionic-Kähler and $\mathcal{M}^{2,1}(M)$ a special Kähler manifold.

1.4 D-branes and instantons

Apart from the fundamental one-dimensional strings, there are other dynamical objects in string theory called D-branes. They can have various dimensionalities and their main property is to serve as boundary conditions for the open strings, i.e. the D-branes are (from zero to ten-dimensional) hypersurfaces in space-time on which the end point of open strings are free to move. A $Dp$-brane in particular is a $p$-dimensional D-brane, which sweeps a $p + 1$ dimensional world-volume when moving in time. D-branes can introduce non-abelian gauge theories in string theory, thus giving a way to reproduce the Standard Model.

Adding D-branes to a type II B configuration leads to a theory that has closed strings in the bulk and open strings ending on the D-branes, thus effectively changing the low-energy field content of the theory. This is why the introduction of D-branes breaks partially the supersymmetry - the open strings form vector multiplets with fewer conserved supercharges, leading to effective decrease in the overall number of supersymmetries. Depending on the way D-branes are embedded they break at least half of the supersymmetries present. Therefore, in the context of Calabi-Yau compactifications of type II B string theory, the best one can hope for after including D-branes is that $\mathcal{N} = 1$ supersymmetry in 4 dimensions holds. At this point it is therefore not yet clear why one should consider such cases. However, in what follows we will see that D-branes are an integral part in the process of string compactifications. Historically D-branes were discovered after the string theory spectra were found and Calabi-Yau compactifications were considered. The absence of D-branes is exactly why it took so long for physicists to be able to find realistic solutions in type II string theories.

D-branes can further carry charges that ensure their stability. This way they can couple electrically and magnetically to the antisymmetric form-fields arising from the R-R string theory spectrum. This works in analogy to the Maxwell theory of electromagnetism. Having an $n$-form gauge field $A_n$, one can define the $(n + 1)$-form field strength $F_{n+1} = dA_n$ that is invariant under gauge transformations $\delta A_n = d\Lambda_{n-1}$ since
\( d^2 = 0 \). In electromagnetism, the field strength is a 2-form that can couple to electric and magnetic sources (although magnetic monopoles have not yet been observed experimentally). With sources, the Maxwell equations become:

\[
\begin{align*}
    dF & = *J_m, \\
    d*F & = *J_e,
\end{align*}
\]

where the sources carry electric and magnetic charges given by

\[
\begin{align*}
    e & = \int_{S^2} *F, \\
    g & = \int_{S^2} F
\end{align*}
\]

(1.33)

with integration over a sphere surrounding the monopoles. This way, the 1-form gauge field couples to point objects (D0-branes in string theory). Generalizing this, \( A_{p+1} \) can couple to a Dp-brane that acts as electric source and to a D\((6 - p)\)-brane as magnetic source with corresponding charges in ten dimensions. Therefore one may think of every Dp-brane as having an electric/magnetic dual D\((6 - p)\)-brane. In analogy to the Dirac quantization condition for electromagnetism, the corresponding D-brane charges must also be quantized:

\[
\mu_{p} \mu_{6-p} = 2\pi n, \quad n = 0, \pm1, \pm2, \ldots
\]

(1.34)

In type II B string theory we saw that there are \( n \)-form fields with \( n = 0, 2, 4 \). Thus, the stable charged Dp-branes that can couple to these fields have \( p = -1, 1, 3, 5, 7 \). The zero form couples electrically to a D\((-1)\)-brane which is an object localized in time and in space, i.e. it makes sense only if interpreted as a D-instanton. Its magnetic dual is a D7-brane. The two-form couples electrically to a D1-brane (D-string) and magnetically to a D5-brane, while the four-form couples both electrically and magnetically to a D3-brane which carries a self-dual charge because the corresponding field strength is also self-dual \( F_5 = *F_5 \). In addition, under some special conditions, one can have stable D9-branes that fill the whole space in Type II B theory. Generally the stable Dp-branes that can ever occur in Type II B are for odd values of \( p \). These stable D-branes are important in that they preserve half of the supersymmetry and are therefore called half-BPS D-branes. In contrast with those, the ”even” Dp-branes in Type II B do not carry conserved charge and are unstable objects that break all the supersymmetry. Due to their instability one loses mathematical control over theories that include these non-BPS branes.

The way to construct the effective actions coming form D-branes is to look at the massless spectrum of the introduced open strings. For a Dp-brane, the corresponding theory that captures the low-energy dynamics is \( p + 1 \) dimensional. We will restrict the discussion to stable D-branes and use world-volume supersymmetry on the brane
to construct the corresponding actions. As in the case of string low-energy effective actions, we will only present the bosonic part of the action and keep in mind the existing supersymmetry that fully determines the fermionic part.

First, the tension of a $D_p$-brane can be determined in terms of the open string coupling constant, which is in turn related to the closed string coupling constant $g_{open}^2 = g_s$. The tension is then

$$T_{Dp} = \frac{1}{g_s(2\pi)^p(\alpha')^{(p+1)/2}}. \quad (1.35)$$

The effective action is determined in static gauge by identifying $p+1$ of the space-time coordinates $X^\mu$ with the D-brane world volume coordinates $\sigma^\alpha$, while the remaining ones are relabeled as $2\pi\alpha'\Phi^i$ to emphasize the fact that they are now scalar fields of the world-volume theory. Then, the effective action in Minkowski space is the Dirac-Born-Infeld (DBI) action:

$$S_{DBI} = -T_{Dp} \int d\sigma^{p+1} \sqrt{-\det(\eta_{\alpha\beta} + k^2 \partial_\alpha \Phi^i \partial_\beta \Phi^i + kF_{\alpha\beta})}, \quad (1.36)$$

with $k = 2\pi\alpha'$ and $F_{\alpha\beta}$ the Maxwell 2-form field strength coming from the open string spectrum vector field $A_\alpha$. We will however need the D-branes to be embedded in curved background coming from the metric $g_{\alpha\beta}$, the NS-NS antisymmetric 2-form $B_{\alpha\beta}$ and the dilaton $\Phi$. Then, the action becomes

$$S_{Dp} = -T_{Dp} \int d\sigma^{p+1} e^{-\Phi_0} \sqrt{-\det(g_{\alpha\beta} + B_{\alpha\beta} + k^2 \partial_\alpha \Phi^i \partial_\beta \Phi^i + kF_{\alpha\beta})}, \quad (1.37)$$

where $\Phi_0$ is the fluctuation of the dilaton around its expectation value $\ln g_s$. The possibility of R-R background fields cannot be captured by the DBI action, but they do contribute to the Chern-Simons term of the corresponding effective action of the string theory. We will not discuss this in more detail, but one needs to keep this in mind for the complete low-energy action of strings and D-branes.

The discussion above considered a single D-brane in space-time. Clearly adding more branes does not just mean that we add the corresponding action to the one that we have, because D-branes can interact with each other via open strings stretching between the different branes. In particular, when $N$ $D_p$-branes coincide, the resulting world-volume theory is a $U(N)$ gauge theory. Therefore one can imagine a set of space-time (here assumed 4-dimensional) filling D-branes arranged such that the resulting theory is $U(1) \times SU(2) \times SU(3)$. This would be a way to reproduce the Standard Model in four-dimensions after compactifying the other 6 on a Calabi-Yau manifold, but it is not yet fully clear how to do this (see e.g. [35] for a recent review). This is however out of the topic of this work and we will thus concentrate on the previous step, i.e. to
consistently compactify the string theory and obtain a dS universe.

D-branes play an important role not only in introducing a new field content and coupling with form fields, but also because they give rise to non-perturbative effects on the internal manifold called instantons. The precise definition of an instanton in quantum field theory is a solution of the classical equations of motion in Euclideanized space-time with a finite action, see e.g. [12]. Physically this means that instantons happen in an instance of time and contribute to the path integral. In string theory instantons can appear in Calabi-Yau compactifications when Euclideanized branes wrap supersymmetric cycles in the manifold [13]. The world volume of Euclideanized Dp-brane has \( p + 1 \) space dimension and lives only at a point in time. It can then entirely wrap some cycle in the internal manifold as the CY three-fold has six spacial dimensions. We still have ordinary Minkowski space in the other four dimensions, i.e. only the world-volume time coordinate is Euclideanized. If the branes wrap around cycles in such a way that supersymmetry is preserved, the corresponding cycle is called supersymmetric. It is exactly those cases that give a finite non-vanishing contribution to some of the physical quantities in string theory. Note that the path integral of these instantons vanishes due to some zero modes as explained in detail in [12], but it is exactly due to these zero modes that one eventually obtains supersymmetric contributions to the superpotential or the Kähler potential. As explained in [29] the counting of zero modes for a specific cycle eventually determines if it is supersymmetric or not. This translates into a nontrivial condition on the given cycle, depending on its dimension. For example (relevant in the following) it turns out that the 4-cycles that satisfy these criteria, admitting D3-brane instantons, are the ones that have an Euler number \( \chi_E = 1 \). However, this condition is relaxed [30, 31] when one considers flux compactifications (to be discussed in the next subsection) and then one has to go and check each cycle separately. Fundamental string worldsheets as well as NS5-branes can also give rise to instantons (NS5-brane is another fundamental object in string theory - it is the magnetic dual to a string). It turns out that fundamental string instantons give rise to non-perturbative \( \alpha' \) corrections to the Kähler potential, while Dp-branes and NS5-branes contribute to the superpotential non-perturbatively also in \( g_s \). As discussed above, the half-BPS Dp-branes in type II B string theory are the odd ones, i.e. the ones wrapping even dimensional cycles on the Calabi-Yau (of which there are only two- and four-dimensional nontrivial cycles for each Calabi-Yau). Generically each brane wrapping a cycle results in a contribution that depends on the cycle’s physical volume and eventually this leads to stabilizing its modulus as we will describe in quite some detail throughout this paper. Thus D1 and fundamental-string instantons can fix the corresponding two-cycles and D3 instantons can fix the supersymmetric four-cycles. In principle then D5 and NS5 instantons should be able to fix the total volume of the
Calabi-Yau as they can wrap around it, but in practice the contributions from these are negligible for reasonably large size manifolds as discussed in [14].

1.5 Fluxes and orientifold projection

As seen above, realistic Calabi-Yau compactifications of string theory to four large dimensions lead to a number of massless scalar (moduli) fields with undetermined vacuum expectation values (vevs). In the low energy four-dimensional effective action there is no potential for these moduli fields. This is undesirable since in nature around us there are no massless scalars. Furthermore the vevs of the moduli fields correspond to different coupling constants, so theory loses physical predictability unless the moduli fields are stabilized to some specific vacuum. For that we need to generate some potential for the moduli and find its minima. The key idea for this is to consider fluxes threading cycles of the internal manifold. When a \((n+1)\)-form field strength \(F = dA\) goes through a nontrivial \((n+1)\) cycle \(\Sigma_{n+1}\) of the CY manifold, it generates a magnetic flux

\[
\int_{\Sigma_{n+1}} F = n. \tag{1.38}
\]

The same field strength can generate an electric flux in a nontrivial \(5-n\) cycle \(\Sigma_{5-n}\) (a CY three-fold assumed):

\[
\int_{\Sigma_{5-n}} *F = m. \tag{1.39}
\]

The fluxes depend only on the homological properties of \(\Sigma_{n+1}\) and \(\Sigma_{5-n}\). Therefore the same \(F\) can generate different fluxes in all cycles of the corresponding dimension (the total number of cycles given by the corresponding Betti number). The point is now that due to the fluxes threading cycles in the compact geometry, there will be some energy cost that will depend on the choice of metric on the manifold \(M\). Effectively this means there will be some potential generated that depends on the moduli space \(\mathcal{M}\), which is exactly what we were looking for. In principle, this potential is given by a standard Maxwell term in a curved background. For each field strength \(F\) the generated potential is

\[
V = \int_M F \wedge *F, \tag{1.40}
\]

where the metric enters in the definition of the 6-dimensional Hodge star.

This simple expression for the moduli potential (the moduli are then derived from the field-strength components) unfortunately cannot lead to a closed form expression since no Ricci flat metric is known explicitly for any Calabi-Yau. We will see shortly how to evade this problem by calculating the potential in a different way. There is an
even bigger complication due to the fact that inclusion of fluxes effectively changes the whole background geometry and introduces a warp factor for the large four-dimensions as well as for the internal manifold. The full ten-dimensional metric in the presence of fluxes looks like

\[ ds_{10}^2 = e^{A(y)} \eta_{\mu\nu} dx^\mu dx^\nu + e^{-A(y)} g_{mn}(y) dy^m dy^n. \]  

(1.41)

Poincaré invariance allows the warp factor \( A(y) \) to depend only on the coordinates of the internal manifold.

In the 1980s, before D-branes "existed", there were no-go theorems for warped compactifications of string theory with fluxes to a Minkowski or de Sitter space-time [15]. If no brane sources are added, the warp factor and fluxes are necessarily trivial. This can be shown starting from the low-energy effective action Eq. (1.2) and compactifying it on the warped Calabi-Yau space. Then, the classical equations of motion cannot be satisfied unless the warp factor \( A(y) \) is constant and the form field \( G_3 \) vanishes \((G_3 = F_3 - \tau H_3)\) where \( F_3, H_3 \) are as defined for the type II B effective action and the dilaton \( \tau \) is a complex modulus \( \tau = C_0 + i e^{-\Phi} \). It later turned out that brane sources can invalidate this no-go theorem as they cancel the undesired positive contribution from the warp-factor and fluxes. One needs to include D3-branes in order to cancel the contributions from the 3-form field strength \( G_3 \). This generalizes for all dimensions - if one wants to switch on fluxes coming from \( F_n \) they have to add \( D_n \)-branes with certain charge. The equation that determines the charge of the branes and ensures that the classical equations are satisfied is called a tadpole-cancellation condition. For the case at hand (with \( G_3 \)) the condition reads (see [17]):

\[ \frac{4\pi}{(2\pi \sqrt{\alpha'})^8 T_3} \int_M H_3 \wedge F_3 + Q_3 = 0, \]  

(1.42)

where \( Q_3 \) is the total charge carried by the D3-brane and \( T_3 \) is the tension (c.f. Eq.(1.35)). In this case due to the fact that \( G_3 \) is imaginary self-dual on the CY (so that the first term is strictly positive) it turns out that only negative charge can induce 3-form fluxes. We will soon see that the charge \( Q_3 \) can also get contributions from other object living on the CY.

This is not the end of the story because one often adds an extra feature in order to make the flux compactifications with branes more stable. We can introduce orientifold planes (O-planes) in Calabi-Yau manifold in order to stabilize the D-brane configurations [16, 40, 41]. This becomes clear if one considers F-theory compactifications on CY 4-folds, since it is equivalent to type II B compactified on 3-folds after orientifolding (cf. [19]). By definition, an orientifold is obtained from one of the type II string theories by performing a projection involving the worldsheet parity operator \( \Omega \). \( \Omega \) swaps the left
and right moving sectors of a closed string and flips the two ends of an open string.

\[
\text{Closed} : \quad \Omega : (\sigma_1, \sigma_2) \mapsto (2\pi - \sigma_1, \sigma_2), \\
\text{Open} : \quad \Omega : (\tau, \sigma) \mapsto (\tau, \pi - \sigma).
\]  

(1.43)

For type II B this can be further combined with a second symmetry operation, \((-1)^{F_L}\), where \(F_L\) is the space-time fermion number in the left-moving sector. Under the composition of these two operators the low-energy effective fields are either even or odd. The even degrees of freedom are kept in the action as they are, while the odd are projected out leading to reduction of the field space by half. On the Calabi-Yau the orientifold operators can be used together with \(\sigma\): a discrete holomorphic isometry that leaves both the metric and complex structure of the manifold invariant. This is important since we do not want to destroy the underlying manifold. As a consequence, the Kähler form is invariant: \(\sigma^*J = J\) (\(\sigma^*\) denotes the pullback of \(\sigma\) to the manifold). For the holomorphic \((3,0)\)-form \(\Omega\) we are free to choose whether \(\sigma^*\Omega^{3,0} = \Omega^{3,0}\) or \(\sigma^*\Omega^{3,0} = -\Omega^{3,0}\). This choice eventually leads to two different kinds of orientifold projection in type II B theory. Formally they are defined as:

\[
\text{O3/O7} : \quad (-1)^{F_L}\Omega\sigma, \\
\text{O5/O9} : \quad \Omega\sigma.
\]  

(1.44)

These are referred to as orientifold planes since their fix-point sets describe planes. To see this one needs to know a bit more about the worldsheet parity projection \(\Omega\). The interested reader can check the explanation of reference [40], section 3.2. For the following it is important just to know, as already indicated, that the first projection in (1.44) leads to 3 and 7 dimensional orientifold planes, while the second one to 5 and 9 dimensional ones. As explained, these planes effectively project out half of the degrees of freedom, including some of the homology elements. For example the Hodge number \(h^{1,1}\) splits into two parts \(h_+^{1,1}\) and \(h_-^{1,1}\), denoting the numbers of even and odd harmonic \((1,1)\)-forms respectively. The dimensionality of the planes nicely matches the dimensionality of the needed D-branes to stabilize the fluxes. This is no coincidence, in fact the full tadpole cancellation condition has to take into account the orientifold planes, i.e. in the case that we presented above in Eq. (1.42) there are also \(\text{O3/O7}\) planes that further ensure stability by contributing to the tadpole cancellation-condition. If we want to add 5-form fluxes we will then need to correspondingly add D5-branes and \(\text{O5/O9}\) planes.

If all these ingredients are properly added together, we finally obtain a consistent string theory flux compactification on a warped Calabi-Yau manifold that can eventually lead to de Sitter solution for the large four-dimensional space-time. It further
turns out that the phenomenologically interesting cases are the ones with $O3/O7$ orientifold planes and 3-form flux $G_3$ as above because the standard Model fields can be only living on the world-volume of D3 and/or D7 branes [43]. This is also more desirable technically since one can obtain $O3/O7$ orientifolds of type II B also as a limit of F-theory compactified on elliptic Calabi-Yau four-folds [19], while this is not possible for the $O5/O9$ case. This technique is commonly used for realistic orientifold examples (e.g. [38]). The resulting theory is a $\mathcal{N} = 1$ supergravity in four dimensions. The warp factor effectively means that the underlying manifold is strictly speaking not a Calabi-Yau manifold any more and the identities described in 1.2 do not hold. However, it has been shown that in the case of type II B flux compactifications the warp factor introduces only a small perturbation for large CY volumes [42] and therefore we will continue our discussion as if the underlying manifold was indeed Calabi-Yau. This is in contrast to type II A theory and is actually the reason why string theorists focus mainly on type II B.

Now we can turn again to the problem of calculating the moduli potential. In $\mathcal{N} = 1$ there is a closed-form formula relating the generated potential to the Kähler potential $K$ and the superpotential $W$, which both appear in the effective action. In general, we can find them from geometric considerations. As discussed above, the Kähler potential fixes the metric on the manifold and therefore we have strong restrictions on its shape. The superpotential on the other hand is generated by the fluxes as derived in [34]. Apart from their standard $\mathcal{N} = 1$ form, both $K$ and $W$ will also get contributions from perturbative and non-perturbative effects like the ones discussed in subsection 1.4, and we will later be able to estimate which ones are relevant for the full moduli stabilization. Thus, at last one can explicitly find the resulting potential $V(K,W)$. For completely explicit calculations clearly we need to fix the internal manifold since this fixes the number and type of moduli fields, but it is possible to go quite far in stabilizing the moduli for a generic manifold as we will see in due course.

Once having a formula for the potential, the next step is to classify the stable minima where the moduli fields acquire vacuum expectation values in analogy to the Higgs mechanism. The way to see if the minimum found correspond to Minkowski, Anti de Sitter, or de Sitter space is quite straightforward. The relevant terms in the low-energy effective action are

$$S_\phi = \int d^4x \sqrt{-g} \left(R + g_{\mu\nu} \partial^\mu \phi \partial^\nu \phi + V(\phi) + ...\right), \quad (1.45)$$

where we assumed for simplicity one modulus $\phi$ (the argument for more moduli remains the same but calculations are more involved). The classical equations of motion lead
to

$$\partial^{\mu}(g_{\mu\nu}\sqrt{-g}\partial^{\nu}\phi) = -\sqrt{-g}\frac{\partial V}{\partial \phi}. \quad (1.46)$$

This has solution $\phi = \phi_0 = \text{const}$ for $\frac{\partial V}{\partial \phi} = 0$ as desired. Furthermore, the Einstein field equation in this case becomes

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -g_{\mu\nu}V(\phi) \mid_{\phi=\phi_0}. \quad (1.47)$$

This is in fact the modified Einstein equation with cosmological constant $\Lambda = V(\phi_0)$, i.e. the value of the potential at the minimum is equal to the cosmological constant. Therefore, in order to find positive cosmological constant corresponding to de Sitter universe, one needs to show the existence of stable moduli potential minima at positive values of the potential. This is essentially the main topic of this thesis - the search and classification of those minima.
Overview of Type II B Moduli Stabilization

In the past few years there has been great research interest in the field of string phenomenology (for a comprehensive review see [17, 18]). After the KKLT scenario [20] for first time suggested a way to obtain stabilized vacua from type II B string theory building on earlier works such as [23], people realized that this could provide the missing link between the theory and the real world. Presently one can find many extensions and improvements of the original idea, the most notable and well established of which is the Large Volume Scenario (LVS) [21, 22]. It builds up on the KKLT solutions by including also corrections to the tree-level supergravity effective action, which were computed in [24, 26]. Up to now the LVS has passed many consistency checks [28], but there are nevertheless many open problems. The stabilization of the Kähler moduli in type II B requires non-perturbative effects which appear only if certain conditions are satisfied [29, 31]. Also the process of uplifting to a Minkowski or de Sitter vacuum is controversial in the sense that unnatural fine tuning is needed to obtain the desired small positive value for the cosmological constant.

Here we will first try to review and discuss the consistency of the KKLT and the LVS models. They propose a method to consistently stabilize some or all of the geometric Kähler moduli. Then in the next section we will try to propose yet another generalization of the LVS, now including also non-geometric Kähler moduli. Based on [32, 33] we would be able to estimate the importance of the world-sheet instantons on the moduli potential in the large volume limit and conclude that non-geometric moduli cannot be simply neglected and their presence can substantially alter the moduli stabilization procedure.

For now, until the end of this section we assume $h^{1,1}_{-1} = 0$, so no non-geometric moduli are present.
2.1 KKLT

The KKLT procedure stabilizes all moduli in a warped type II B flux compactification on a Calabi-Yau O3/O7 orientifold. The basic assumption here is the neglect of the warping, treating the internal manifold as a standard CY threefold. At tree level, in four dimensional $\mathcal{N} = 1$ supergravity, the Kähler potential is (see e.g. [42])

\[ K = -\ln[i \int_{\text{CY}} \Omega(z) \wedge \bar{\Omega}(\bar{z})] - \ln(-i(\tau - \bar{\tau})) - 2 \ln(V_{\text{CY}}), \quad (2.1) \]

where the first term depends only on the complex structure moduli $z$, the second on the dilaton field $\tau$, and the third one implicitly depends on the Kähler moduli via the volume $V_{\text{CY}}$ of the CY manifold as in (1.29). We need to use the 4-cycle volumes $\tau_\alpha = \frac{1}{2} \kappa_{\alpha\beta\gamma} v^\beta v^\gamma$ as Kähler moduli basis since the metric obeys (1.8) only in this case (cf. [42]). The superpotential at tree level is independent of the Kähler moduli and is given by the famous Gukov-Vafa-Witten [34] flux superpotential

\[ W = \int_{\text{CY}} \Omega(z) \wedge (F_3 - \tau H_3). \quad (2.2) \]

A detailed calculation of the Kähler potential and the superpotential was carried out in the Appendix of [42], both follow from the $\mathcal{N} = 2$ dimensional reduction of the low-energy effective action before orientifolding. The standard $\mathcal{N} = 1$ F-term potential for the moduli fields generated from (2.1) and (2.2) is given by

\[ V = e^K \left( K^{IJ} D_I W D_J \bar{W} - 3|W|^2 \right). \quad (2.3) \]

Here, the indices $I, J$ run over all moduli fields, the matrix $K^{IJ}$ is the inverse of $K_{IJ} = \partial_I \partial_J K$, and $D_I W = \partial_I W + \partial_I K \cdot W$. The potential at three-level has the peculiar feature that the sum over the Kähler moduli is exactly canceled by the term $3|W|^2$ as shown explicitly in Appendix B.1. Then,

\[ V_{\text{no-scale}} = e^K \left( K^{A\bar{B}} D_A W D_{\bar{B}} \bar{W} \right), \quad (2.4) \]

where the sum now goes only over the complex structure moduli and the dilaton. The potential is therefore flat in the directions of the Kähler moduli and one can only find minima of the potential with respect to the $z$ and $\tau$ fields. At this point the procedure introduced by KKLT is the following:

- stabilize the complex structure and dilaton moduli at a supersymmetric minimum of the potential (2.4), s.t. $D_z W = D_\tau W = 0$ (the solutions of these equations are a thesis topic by its own and can be dealt with statistically as done in [27]).
• introduce non-perturbative correction to the superpotential in order to generate potential in the direction of the Kähler moduli,

• find minima of the new potential under the initial assumption that the complex structure and dilaton moduli are already fixed and that one can fine tune the fluxes.

The instanton correction of the superpotential can arise from euclideanized D3-branes wrapping four-cycles in the internal manifold. The superpotential (in one-instanton approximation neglecting multi-instanton subleading contributions) reads

\[ W = W_{\text{tree}} + W_{np} = W_0 + \sum_i A_i e^{-a_i \rho_i}, \]  

(2.5)

where \( W_0 \) is now constant - it is the value of the GVW-superpotential at the supersymmetric minimum. The \( A_i \)'s are determinants that appear as prefactors to the instanton corrections, they depend in a nontrivial and not yet fully understood way on all moduli except the Kähler ones. The \( a_i \)'s are numerical constants of order 1 that depend on the specific cycle. The above listed procedure means that we require \( D_{\rho_i} W = 0 \) to hold.

The instanton corrections depend on the complexified Kähler moduli \( \rho_i = \tau_i + i b_i \), where \( \tau_i \) are the volumes of the 4-cycles that the D3-branes wrap around and \( b_i \) are axion fields arising from the integral of the RR 4-form over the corresponding 4-cycles (cf. eq. (1.27)). Note that the volume of the CY is only a function of the \( \tau_i \)'s, \( V_{\text{CY}} = V_{\text{CY}}(\tau_i) \). To make the discussion more explicit, let us take the original KKLT proposal and assume only one Kähler modulus \( \tau_1 \) with corresponding \( b_1 = 0 \) (s.t. \( \rho_1 = \tau_1 \)). The condition \( D_{\tau_1} W = 0 \) gives the equation

\[ W_0 = -A e^{-a_1 \tau_1} \left( 1 + ka_1 \tau_1 \right), \]  

(2.6)

with \( k \) a constant that depends on the exact normalization conventions for the volume of the CY manifold (\( k = 2/3 \) in the original paper [20]). Since \( W_0 \) is already a fixed constant after the stabilization of the complex structure and dilaton moduli, the above equations determines exactly the value of \( \tau_1 \). At this point, the resulting minimum of the potential from Eq. (2.3) is just

\[ V_{\text{min}} = -3e^K W^2 = -\frac{a_1^2 A_1^2 e^{-2a_1 \tau_1}}{6 \tau_1}. \]  

(2.7)

Clearly the potential is negative and the value of \( \tau_1 \) will be finite, typically around 100 (depending on the exact values of \( a_1, A_1, W_0 \)). Note that \( W_0 \) need to be a small negative number in order for the procedure to lead to a minimum and not a maximum of the potential. This can be generalized for the case of more than one Kähler moduli, leading to some more complicated expression for the potential. In such a case even
fine-tuning of $W_0$ might not be sufficient to ensure a minimum in all directions of the moduli space.

The important result is that now the full potential (2.3) depends explicitly on the Kähler moduli, i.e. the no-scale structure is broken. As each term in $W_{np}$ is exponentially suppressed in some Kähler modulus, so will be the resulting terms in the potential. This is however not consistent with our neglect of $\alpha'$ and $g_s$ corrections that generally appear in addition to the tree-level effective action. Since these corrections go as some powers of the Kähler moduli $\tau_i$ they will dominate the exponentially suppressed terms coming from the non-perturbative superpotential. Therefore their neglect can be justified only if the complex structure and dilaton moduli are stabilized at a very small value for $W_0$. This assumption is valid for a very restricted parameter space and does not represent the generic case we eventually desire. Another problem, pointed out in [36], is that the minimum found in the Kähler moduli space following the KKLT procedure does not have to be a full minimum of the potential as it might only be a saddle point in the complete moduli space. We will discuss this issue in detail when we compare the KKLT scenario with the LVS. Nevertheless, in some specific cases and models the KKLT procedure does lead to a stable anti-deSitter minimum [37, 38].

The last step of the KKLT procedure is to uplift the obtained AdS vacuum to a metastable dS vacuum with very small positive value of the potential (i.e. small cosmological constant). This is performed by adding effects from anti-branes. $\bar{D}3$-branes have to be added in order to satisfy the tadpole condition if one puts additional flux, the resulting contribution to the potential is of the form:

$$
V_{\bar{D}3} = \frac{\sigma V_{CY}}{\sqrt{V_{CY}}}.
$$

After suitable fine tuning of the coefficient $\sigma$ one can ensure that the minimum is long lived (very low tunneling probability) and is stabilized at the desired very small value of the cosmological constant. However, the procedure requires a very specific value for $\sigma$ and furthermore assumes all moduli stabilized before introducing the $\bar{D}3$-term, which strictly speaking is incorrect since (2.8) is of course also moduli-dependent.

Now we move on to discuss the Large Volume Scenario where some of the mentioned subtleties are taken care of and more corrections are taken into account.

### 2.2 LVS

The LVS goes along the lines of KKLT, with the difference that now we also consider $\alpha'$ corrections to the Kähler potential, as first suggested by [24, 25] using mirror symmetry considerations. Furthermore, completely generic fluxes are taken into account, i.e. the
condition $D_I W = 0$ is dropped (leading to a loss of supersymmetry). Therefore in the end the major difference between the KKLT and LVS procedures is that one leads to AdS supersymmetric minima, while the other to AdS non-supersymmetric minima, as will be explained in detail in the end of the subsection. The modified version of (2.1) is

$$K = K_{CS} - \ln(-i(\tau - \bar{\tau})) - 2 \ln(V_{CY} + \frac{\xi}{2}\left(\frac{\tau - \bar{\tau}}{2i}\right)^{3/2}),$$

where $\xi$ is constant, proportional to the Euler number of the CY manifold:

$$\xi = -\frac{\chi \zeta(3)}{2(2\pi)^3},$$

and $K_{CS}$ is the complex structure Kähler potential at tree level as given in the previous subsection. Now, in order to calculate the moduli potential (2.3) one has the nontrivial problem of calculating $K_{IJ}$ by inverting $\partial_I \partial_J K$. In this case a well-known complete analytic solution exists [24, 26]. However, we repeat the calculation in detail in Appendix B.1, as this step turns out to be of crucial importance in the following section.

These $\alpha'$-corrections break the no-scale structure of the potential even without the need of non-perturbative effects. As discussed above they contribute to the potential as a power of the volume since:

$$-2 \ln(V_{CY} + \frac{\xi}{2}\left(\frac{\tau - \bar{\tau}}{2i}\right)^{3/2}) = -2 \ln V_{CY} - 2 \frac{\xi}{V_{CY}}\left(\frac{\tau - \bar{\tau}}{2i}\right)^{3/2} + ....$$

Using the same superpotential (2.5) as in the KKLT scenario with the new form of $K$, (2.3) leads to the structure

$$V = e^K[V_{np1} + V_{np2} + V_{\alpha'}]$$

with

$$V_{np1} = K^{ij}\partial_i W_{np}\partial_j \bar{W}_{np},$$

$$V_{np2} = K^{ij}[\partial_i W_{np}\bar{K}_j(\bar{W}_0 + \bar{W}_{np}) + K_i(W_0 + W_{np})\partial_j \bar{W}_{np}],$$

and

$$V_{\alpha'} = (K^{ij}K_i\bar{K}_j - 3)|W|^2.$$
Kähler moduli coming from the geometric moduli \( v^\alpha \) and the non-geometric \( b^\alpha \)'s as explained in section 1 (here again the \( b^\alpha \)'s are zero). In particular, \( \rho_\alpha \) are defined in terms of the 4-cycle volumes \( \tau_\alpha \) derived from the 2-cycles \( v^\alpha \). The two homologies are equivalent so one can consistently change the basis of moduli this way. In terms of the 2-cycle volumes, the 4-cycle volumes are given by \( \tau_\alpha = \frac{1}{2} \kappa_{\alpha\beta\gamma} v^\beta v^\gamma \). As explained in details (with specific examples) in Appendix A, this change of basis can sometimes be very cumbersome calculationally, but nevertheless always possible in principle.

As shown in detail in App. B.1 and in [21] it is relatively simple to obtain the full analytic expression for the full moduli potential (2.12). Unfortunately this fact alone is not very illuminating in our search for stable minima since the potential is a function of many variables and one has to make sure that the special points of interest are indeed minima with respect to all variables and not just saddle points. Furthermore we need to be consistent with our underlying approximation that the volume of the CY manifold is large enough. This translates into a nontrivial condition on the Kähler moduli that depends on the specific choice of the manifold (the volume is an implicit function of the four-cycle volumes). In order to make the situation easier to handle and to obtain a clear condition on the parameter range [21, 22] developed the Large Volume Scenario. It states from the beginning that we concentrate only on the solutions at large volume (in natural units this would mean \( V_{CY} > 10^4 - 10^5 \)) and therefore take only the leading terms of (2.13), (2.14), and (2.15) in this limit. Furthermore, a ”Swiss cheese” type of manifold is assumed [22], which means that the large volume of the manifold is obtained by having one big four-cycle \( \tau_b \) and the remaining ones are stabilized at relatively small values (i.e. \( g_s << \tau_s << \tau_b \) such that the one-instanton approximation is valid). This is possible because \( \frac{\partial^2 V_{CY}}{\partial v^\alpha \partial v^\beta} = \kappa_{\alpha\beta\gamma} v^\gamma \) has a signature \((1, h^{1,1} - 1)\) for every Calabi-Yau [22]. Applying change of variables from \( v^\alpha \) to \( \tau_\alpha \) gives analogously that \( \frac{\partial^2 V_{CY}}{\partial \tau_\alpha \partial \tau_\beta} \) has signature \((1, h^{1,1} - 1)\). This means that when chosen in suitable basis, only one of the 4-cycles contributes positively to the volume, and all others tend to decrease it. Thus a large volume can be obtained by high value of this specific 4-cycle volume and small volumes for all other \( \tau_\alpha \)'s. It is not clear how generic this is and whether one can always find such a suitable basis for every manifold. The very simple examples from Appendix A show that one can have very complicated dependence of \( V_{CY} \) on the \( \tau_\alpha \)'s already for values of \( h^{1,1} \) as low as 2 or 3. One can then imagine that when \( h^{1,1} \sim O(100) \) things go out of control. However, let us assume for the moment that the manifold is nice enough, since this is crucial to proceed with our task. Then \( V_{CY} \approx \tau_b^{3/2} \) and so we obtain an explicit volume dependence in the moduli potential. At this point the potential is still a function of the same number of variables as before as we just traded \( \tau_b \) for \( V_{CY} \), so the problem of finding full minima remains.
Let us now concentrate on a specific example which we will try to extend to the general case afterwards. Assume that we are dealing with a CY manifold with only 2 Kähler moduli, i.e. two cycles - one big \( \tau_b \) and one small \( \tau_s \). An explicit realization of this is the \( \mathbb{P}^4_{1,1,1,6,9} \) model, first presented in [38]. The volume of the manifold in this case is

\[
V_{\text{CY}} = \frac{1}{9\sqrt{2}} \left( \tau_b^{3/2} - \tau_s^{3/2} \right).
\] (2.16)

The potential in the large volume limit takes the form

\[
V = e^{K_{CS}} \left[ -\frac{\alpha \tau_s e^{-a_s \tau_s}}{V_{\text{CY}}^2} + \frac{\beta}{V_{\text{CY}}^3} + \frac{\gamma \sqrt{\tau_s e^{-2a_s \tau_s}}}{V_{\text{CY}}} \right].
\] (2.17)

The coefficients \( \alpha, \beta, \) and \( \gamma \) depend in general on the complex structure and dilaton moduli and their exact expressions are well-known (cf.(30)-(32) of [21] and (56)-(58) of [28]). For our discussion at the moment the precise formulae will be unimportant. We just note that \( \alpha \) and \( \gamma \) are assumed in literature to be strictly positive, while the sign \( \beta \) depends on the Euler number of the CY manifold. A positive \( \beta \) is obtained from negative Euler number and vice versa. It is also important to note how the coefficients scale generally with the dilaton field \( S_1 = (\frac{\tau - \bar{\tau}}{2i}) \) (as introduced in App. B.1) and the tree-level superpotential \( W_0 \):

\[
\alpha \sim \frac{|W_0|}{S_1}; \quad \beta \sim \sqrt{S_1 |W_0|^2}; \quad \gamma \sim \frac{1}{S_1}.
\] (2.18)

Note that the leading terms in the exponents of the potential in (2.17) do not depend explicitly on \( \tau_b \) as the exponents containing it are exponentially suppressed compared to the ones with \( \tau_s \).

Therefore we can now investigate the potential as a function of the two variables \( V_{\text{CY}} \) and \( \tau_s \) (we will drop the subscript in the following as it is redundant). Until now the standard way to do that was to state that \( \tau \sim \ln V_{\text{CY}} \) and effectively reduce the problem to a one-dimensional one, classifying the possible minima afterwards. This method leads to AdS minima of the potential given that \( \beta \) is positive number of order 10-100 times the values of \( \alpha \) and \( \gamma \). Let us take slightly more quantitative approach. Clearly we know that \( \tau \) must be indeed small compared to \( V_{\text{CY}} \) as this is our main assumption in the large volume scenario. We can then introduce a numerical coefficient \( n \), such that

\[
a \tau = n \ln V_{\text{CY}}.
\] (2.19)

This means that \( e^{-a \tau} = V_{\text{CY}}^{-n} \) and the full potential reads

\[
V = e^{K_{CS}} \left[ -\frac{\alpha' \ln V_{\text{CY}}}{V_{\text{CY}}^{2+n}} + \frac{\beta}{V_{\text{CY}}^3} + \frac{\gamma' \sqrt{\ln V_{\text{CY}}}}{V_{\text{CY}}^{1+2n}} \right].
\] (2.20)
Here the coefficients $\alpha'$ and $\gamma'$ are slightly different from $\alpha$ and $\gamma$ due to the presence of $a$ and $n$, but this is irrelevant for our discussion as they do not change sign or scaling order. In the following we will treat these coefficients as tunable parameters to see the range which allows for large volume minima, but one should keep in mind that in fact they are completely determined after fixing the complex structure and dilaton moduli. Now we are interested in the form of the potential depending on the numerical factor $n$. Looking at the formula we see that the denominator of the three terms is the same only at $n = 1$. We will then distinguish between three main classes of values for $n$.

1. $n > 1$:
   Since $3 < 2 + n < 1 + 2n$ we have that the leading term at large volumes in (2.20) is the term with the $\beta$ coefficient. The most suppressed term, which dominates at small volumes is the one with $\gamma'$. Naively speaking, if we can freely choose the coefficients we would obtain minima in two cases. If both $\beta$ and $\gamma'$ are positive (i.e. the potential starts at large positive value and ends up asymptotically at positive zero) we need to have $\alpha'$ large enough to dominate in the intermediate volume range, ensuring that minima can be obtained. Furthermore fine tuning of $\alpha'$ would in principle allow us to have both AdS and dS minima. If $\beta$ is negative, then the presence of minima is ensured for any value of $\alpha'$ since the potential starts at large positive value and crosses the zero somewhere to finally end at negative zero. This means that somewhere on the way there must be an AdS minimum.

2. $n = 1$:
   Here the power of $V_{\text{CY}}$ in the denominator of all three terms is the same so the logarithmic dependence in the numerators becomes important. This case is well-known and extensively discussed in literature and we know that AdS minima exist for positive $\beta$ since $\alpha'$ is also positive and the situation resembles the second case of the above class of minima as the potential starts at large positive number and ends at negative zero. Here the negative $\beta$ leads only to presence of maxima in the potential and no minima are possible.

3. $n < 1$:
   In this case $1 + 2n < 2 + n < 3$ so the leading terms is $\gamma'$ and the most suppressed one is $\beta$. Again the choice of negative $\beta$ will lead only to maxima since the potential will start at large negative value. However, for $\beta > 0$ we have a large variety of possibilities depending on $\alpha'$ since here the function starts positive and ends positive as in the first case of the first class. For small $\alpha'$ there are no minima, for large $\alpha'$ there are AdS minima, and for intermediate fine tuned values we can even obtain dS minima.
So now we have a full classification of the possible minima in the direction of the Kähler moduli in our model potential (2.20). Unfortunately this is far from enough to draw any final conclusions on how to consistently stabilize the moduli. In the above we did not discuss at all if the minima are at large volumes or not, we just classified the cases and the type of minima. In order to have a self-consistent solution we must concentrate only on the large volume minima and for this purpose we have to examine more closely the numerical behavior of the coefficients $\alpha', \beta$, and $\gamma'$, as well as $n$.

First thing to note is that the three terms in (2.20) need to be really competitive to each other until large volumes as otherwise the extrema will be pushed to the left and everything at large volumes will depend on one term only. This fact rules out a huge range of possibilities or $n$. If $n$ is too big the leading $\beta$ will win over the other terms at volumes of order 1. Plotting numerically the potential with $n > 1$ further assures us that the values we are interested in cannot go higher than $n \approx 1.1$. On the other hand, small $n$ would mean dominance of the $\gamma'$ term excluding the possibility of large volume extrema. Therefore we also have to restrict to values bigger than $n \approx 0.85$. So the range of interest for us is $0.85 < n < 1.1$.

The fact that we narrowed down the possible values of $n$ does not yet assure us that we have found large volume minima, it just means that these can certainly not exist outside of this range. Let us now discuss again the three different classes of solutions one by one taking into account the numerical values of $\alpha'$, $\beta$ and $\gamma'$.

1. $1 < n < 1.1$:
   Here it turns out that $\gamma'$ needs to be an order of magnitude (10-20 times) larger than $\alpha'$ and $\beta$ to make sure that this term still plays a role. This condition is enough to ensure large volume AdS minima for both positive and negative values of $\beta$ as shown on Fig. 2.1 and 2.2. In the latter case the AdS minimum is followed by a dS maximum as expected since the potential crosses the zero again after the AdS minimum. Therefore decreasing and very fine tuning the $\alpha'$ parameter allows us (still in the case where $\beta > 0$, i.e. negative Euler number) to obtain a dS minimum as shown on Fig. 2.3. In this case the value and position of the minima can be changed by changing the relative weight between $\alpha'$ and $\beta$, but we always require that $\gamma'$ is substantially bigger. As discussed in detail in the following this can in principle be realized, but is not very probable as we require $W_0$ to be very small.

2. $n = 1$:
   This case is well described in literature and we will just review the needed conditions. Here the logarithmic behavior with $V_{CY}$ is more important and at large
Figure 2.1: (unstable) AdS minimum at $V_{CY} \approx 1.08 \times 10^{11}$ with $n = 1.05$, $\alpha' = 1$, $\beta = -1$, and $\gamma' = 20$ (Positive Euler number $\chi_E$).

Figure 2.2: AdS minimum at $V_{CY} \approx 7.56 \times 10^{12}$ with $n = 1.05$, $\alpha' = 1$, $\beta = +1$, and $\gamma' = 20$. 
volumes we need $\beta$ to be far bigger (20-100 times) than $\alpha'$ and $\gamma'$ so that the terms comparable for big $V_{CY}$. In fact it is usually argued that this is very plausible since $\beta$ scales as square root of the inverse string coupling constant, while $\alpha'$ and $\gamma'$ are proportional to it. At weak coupling therefore we expect $\beta$ to be bigger. This reasoning is strengthened if we also take big $W_0$ as $\beta$ scales as higher power with $W_0$ compared to $\alpha'$ and $\gamma'$.

3. $0.85 < n < 1$:

This class of large volume minima can be obtained similarly to the the above case, i.e. by requiring $\beta$ to be substantially larger. Here $\beta$ has to be positive and $\alpha'$ has to be big enough as otherwise the potential will not exhibit any minima. What "big enough" means is determined by the value of $n$, but typically $\alpha'$ and $\gamma'$ can be of the same order. On Fig. 2.4 one can see a typical AdS minimum where $\alpha' = 1.2$ compared to $\gamma' = 1$ (note that only the relative weight between the constants matters and a constant rescaling of all of them will not lead to any difference). Having found an AdS minimum, it is clear that we can also have a dS minimum at the same value of $n$, $\beta$ and $\gamma'$ if we are allowed to fine tune $\alpha'$ - we just decrease it slowly until we have that the minimum is slightly above zero. For the same example we see that $\alpha' = 1.1885$ leads to such dS minimum as shown on Fig. 2.5. Further decrease of $\alpha'$ will wipe out the minimum entirely, so there is only a very small range of allowed values for $\alpha'$ that leads to dS vacua when we fix all other constants. Nevertheless, this possibility is real and we do not need to introduce anti-branes or any other mechanism for uplifting from AdS to dS minima.

Figure 2.3: (unstable) dS minimum at $V_{CY} \approx 2.46 \times 10^{24}$ with $n = 1.05$, $\alpha' = 0.45616$, $\beta = +1$, and $\gamma' = 20$, (plot crosses 0 for higher $V_{CY}$).
Figure 2.4: AdS minimum at $V_{CY} \approx 3.52 \times 10^{12}$ with $n = 0.95$, $\alpha' = 1.2$, $\beta = 50$, and $\gamma' = 1$.

Figure 2.5: (unstable) dS minimum at $V_{CY} \approx 2.59 \times 10^{13}$ with $n = 0.95$, $\alpha' = 1.1885$, $\beta = 50$, and $\gamma' = 1$, (plot crosses 0 for higher $V_{CY}$).
We therefore see a lot richer variety of possible vacua even in this very simple case of two Kähler moduli. So far only case 2 was seriously discussed in [21, 22] and the consequent articles on the subject. The third case where $n$ is slightly less than 1 was not discussed but exact numerical solutions such as the one in the appendix of [28] imply it.

Going back from where we started - Eq. (2.20) has to be minimized with respect to both $n$ and $V_{CY}$. We described in details the minima in the volume direction at fixed $n$. To obtain the full minima in all directions we just have to imagine the graphs with different $n$ put one next to another for continuous values of $n$, a situation resembling slices of bread put together to form the initial shape again. Putting the two-dimensional figures together we get back the full potential depending on the two parameters $V_{CY}$ and $n$, which is equivalent to potential depending on $\tau_b$ and $\tau_s$.

For the case when $\beta > 0$ we see that starting from big $n$ slice the potential rolls to zero at $V_{CY} \to \infty$. As $n$ comes near to 1 we start having dS minima at finite volume and at the point where $n = 1$ we will for sure have an AdS minimum and then decreasing $n$ will lead to continuous rise of the potential at the finite volume minimum. Therefore we see a minimum in the $n$ direction which corresponds to the AdS minimum of the potential, i.e. $n$ will take a value that ensures $V < 0$. The exact value of $n$ could be slightly above or below 1 depending on $\alpha$ and $\gamma$, and the minimum will be at large volumes only for the cases discussed above (either $\beta \gg \alpha \approx \gamma$, or $\gamma \gg \alpha \approx \beta$). One can see examples for the realization of these cases on Fig. 2.6 and 2.7.

For the case when $\beta < 0$ we have large volume minima only for $n > 1$ and then the minima appear at small volumes with larger negative value for decreasing $n$. Therefore, in this case there is no possibility for stable minima in both directions that are consistent with the approximation for large volume. In short, we have reached the same conclusion as in [21, 22] in a bit more systematic and rigorous way. Unfortunately the sign of $\beta$ indeed turns out to be very important in our procedure, which is undesirable generically.

The most reasonable question at this point is how would this generalize to the case with many small four-cycles. Well, we can imagine that for each one of them we will have a different proportionality constant $n$. Therefore we will end up with a polynomial function of $V_{CY}$ parametrized by the initial constants $\alpha$, $\beta$ and $\gamma$, and now by all constants $n_1, ..., n_k$ for $k$ small Kähler moduli. Therefore we will obtain even more possibilities of obtaining different extrema. However, the generic behavior of the potential will still be determined to a large extent by $\alpha$, $\beta$ and $\gamma$, their sign and relative weight. For example, if we take a model with two small four-cycles and $n_1 = 0.95$, $n_2 = 1.05$ we can obtain both large volume minimum and large volume maximum for a suitable
Figure 2.6: AdS minimum for three Kähler moduli at $V_{\text{CY}} \approx 1.71 \times 10^7$ with $n = 1.136$, $\alpha' = 1$, $\beta = 1$, and $\gamma' = 20$.

Figure 2.7: AdS minimum (potential multiplied by $10^{40}$) at $V_{\text{CY}} \approx 4 \times 10^{18}$ with $n = 0.969$, $\alpha' = 1.585$, $\beta = 155$, and $\gamma' = 1.066$. 
Figure 2.8: AdS minimum for three Kähler moduli at $V_{CY} \approx 3.05 \times 10^9$ with $n_1 = 0.95$, $n_2 = 1.05$, $\alpha' = 1$, $\beta = 30$, and $\gamma' = 1$.

Figure 2.9: dS maximum at $V_{CY} \approx 2.41 \times 10^{13}$ with $n_1 = 0.95$, $n_2 = 1.05$, $\alpha' = 1$, $\beta = 30$, and $\gamma' = 1$. 
choice of $\alpha$, $\beta$ and $\gamma$ as shown on Fig. 2.8 and 2.9. Here we again need to have $\alpha$ big enough compared to $\gamma$ similarly to third case above. Fine tuning of $\alpha'$ once again leads to a dS minimum - Fig. 2.10. Adding more small cycles will lead to more extrema and we lose control to predict the exact shape. Nevertheless, one can imagine that all cases discussed above can happen again for more moduli.

In general the more directions (i.e. moduli) there are, the more possibilities will there be to roll down to the negative minimum which always appears when $n_1 = n_2 = \ldots = 1$ (for $\beta > 0$). Therefore we cannot expect any local dS minima, and the generic results for the two moduli case will still hold. For the same reasons we cannot expect new large volume minima to appear when $\beta < 0$.

Minimizing Eq. (2.12) at first sight does not mean we have fully minimized the full potential in all directions. Recall that the complex structure and dilaton moduli were assumed to be stabilized at values such that $D_{\tau_i} W = D_{\tau} W = 0$, leading to a set of coupled equations that can be solved in principle. These equations do depend on the Kähler moduli because of the non-perturbative superpotential and the mixing of the dilaton and the Kähler moduli in the Kähler metric. Therefore we should have substituted the values for $\tau$ back in (2.12) where it appears only as constant. Since we do not know the explicit equations, one clearly loses control of how exactly the dilaton depends on the $\tau_i$ and therefore at this point we need to assume that at large volume the dependence is negligible and $\tau$ is fixed to a constant. This is a justified approximation since in the Kähler metric $K_{\tau}$ goes as $V_{\text{CY}}^{-1}$ and the non-perturbative superpotential is exponentially suppressed in the $\tau_i$’s. In this case, the full potential in

Figure 2.10: (unstable) dS minimum at $V_{\text{CY}} \approx 1.57 \times 10^{11}$ with $n_1 = 0.95$, $n_2 = 1.05, \alpha' = 0.96617$, $\beta = 30$, and $\gamma' = 1$. 
terms of $D_z W$ and $D_z \bar{W}$ is given by:

$$
V = e^K (K^{\alpha \beta} D_\alpha W D_\beta \bar{W} + K^{\tau \bar{\tau}} D_\tau W D_{\bar{\tau}} \bar{\bar{W}}) \\
+ \frac{\xi}{2V_{CY}} e^K (WD_\tau \bar{\bar{W}} + \bar{W} D_\tau W) + e^K [V_{\alpha'} + V_{\nu'1} + V_{\nu'2}].
$$

(2.21)

The first term is positive semi-definite and is only zero at the supersymmetric case $D_\alpha W = D_{\bar{\tau}} W = 0$. This term dominates the other two at large volumes as it scales as $V_{CY}^{-2}$ while the two others scale as $V_{CY}^{-3}$. Therefore any movement of the complex structure and dilaton moduli away from the supersymmetric point increases the potential, i.e. this point is a stable minimum and finding the minima of the third term with respect to the Kähler moduli ensures a full minimum of the potential. This is indeed the situation described above and therefore the minima that were found automatically satisfy the required property.

It is interesting to note that the KKLT procedure of finding supersymmetric minima in all directions may not lead to full minima by the same argument. Putting explicit volume dependence, in the KKLT scenario

$$
V = e^{K_{cs}} \left( \frac{K^{ij} D_i W D_j \bar{W}}{V_{CY}^2} - \frac{3|W|^2}{V_{CY}^2} \right).
$$

(2.22)

Here both terms scale with volume in same way. The first term is positive semi-definite as before, but now moving away from the supersymmetric point may change the overall prefactor $e^{K_{cs}}$ in a way that the full potential decreases, even though the first term contributes some small positive value. Therefore the KKLT procedure of stabilizing all moduli at the supersymmetric point might also lead to maxima and saddle point apart from minima and one has to check case by case the outcome. Note that in any case the KKLT procedure will lead to extrema of the potential. This is easy to see from calculating the first derivative of the potential with respect to a given field. Keep in mind that the superpotential is a holomorphic function and therefore either $W$ or $\bar{W}$ will depend on a given modulus but never both. Let us then take (without loss of generality as the other case is identical) a holomorphic modulus $m$ and evaluate its derivative at the supersymmetric point where $V = -3e^K W \bar{W}$.

$$
\frac{\partial V}{\partial m} = -3K_m e^K W \bar{W} - 3e^K \partial_m W \bar{W} = \\
-3e^K W (\partial_m W + K_m W) = -3e^K W D_m W = 0.
$$

(2.23)

So, once having solved the coupled equations for $D_\tau W = 0$ we are ensured to find an extremum of the potential. However, explicit solution of these equations can be given only after choosing a specific Calabi - Yau manifold and any generic behavior of the
solutions is very hard to find. Therefore the LVS scenario, although explicitly breaking
the supersymmetry, leads to a higher level of control over the solution and possibility
to find explicitly large volume minima in certain cases.
The previous scenarios, as well as the vast literature on the subject of type II B moduli stabilization, focus the attention on stabilizing the Kähler moduli at a value where the internal manifold has a large volume as consistency requires. However, in this process the non-geometric Kähler moduli are usually completely disregarded and assumed non-existent. This is only justified in special cases for orientifold projections where \( h_{(1,1)}^- = 0 \), as otherwise we have additional moduli coming from the 2-form fields \( \hat{B}_2 \) and \( \hat{C}_2 \) of the type II B low energy effective action:

\[
\hat{B}_2 = b^a(x)\omega_a, \quad \hat{C}_2 = c^a(x)\omega_a, \quad a = 1, \ldots, h_{(1,1)}^+, \tag{3.1}
\]

where \( \omega_a \) is the basis of harmonic \((1,1)\) forms which are odd under the orientifold projection. These additional moduli enter the moduli space together with the surviving two-cycle volumes coming from the decomposition of the Kähler from \( J \):

\[
J = v^\alpha(x)\omega_\alpha, \quad \alpha = 1, \ldots, h_{(1,1)}^+, \tag{3.2}
\]

where now \( \omega_\alpha \) are the harmonic \((1,1)\) forms that are even under the orientifold projection. A more detailed calculation and explanation about the orientifold projection and the resulting effective action is given in [39, 40]. Here we will just summarize their results about the resulting Kähler coordinates, giving rise to the correct Kähler metric. We will additionally need the moduli coming from the 4-form \( \hat{C}_4 \) with a decomposition:

\[
\hat{C}_4 = D_2^\alpha(x) \wedge \omega_\alpha + V^\kappa(x) \wedge \alpha_\kappa + U_\kappa(x) \wedge \beta_\kappa + \rho_\alpha(x)\tilde{\omega}^\alpha, \quad a = 1, \ldots, h_{(1,1)}^+, \tag{3.3}
\]

with \( \tilde{\omega}^\alpha \) the basis of harmonic \((2,2)\)-forms, dual to the \((1,1)\) basis that is even under the orientifold projection.

With these definitions, the Kähler metric is given in terms of the reduced complex structure coordinates in the canonic way coming from the explicit manifold and in
terms of the dilaton and the Kähler coordinates that are arranged as follows [46, 39]:

\[
\begin{align*}
\tau &= C_0 + i e^{-\phi}, \\
\kappa_a &= c_a - \tau b^a, \\
T_\alpha &= \frac{3i}{2} \rho_\alpha + \frac{3}{4} \kappa_\alpha (v) + \frac{3i}{4(\tau - \bar{\tau})} \kappa_{\alpha\beta} G^\beta (G - \bar{G})^\alpha, \\
\kappa_{\alpha\beta\gamma} &= \kappa_{\alpha\beta} v^\gamma = 6 V_{\text{CY}}.
\end{align*}
\]

where \( \kappa_\alpha \equiv \kappa_{\alpha\beta\gamma} v^\beta v^\gamma \), i.e. it is just a four-cycle volume with a different normalization. In this notation, \( \kappa = \kappa_{\alpha} v^\alpha = 6 V_{\text{CY}} \). The numbers \( \kappa_{\alpha\beta\gamma} \) and \( \kappa_{\alpha\beta\gamma} \) are the usual Calabi-Yau intersection numbers after performing the orientifold projection. As explained in [39, 32], in the process of orientifolding consistency requires that only the intersection numbers with even number of Latin indices are non-zero. This means that for all \( \alpha, \beta, a, b, c \), \( \kappa_{\alpha\beta a} = \kappa_{abc} = 0 \) has to hold.

Having this, the Kähler potential is the same as before, i.e. (2.1) at tree-level and (B.16) with \( \alpha' \)-corrections. From the above definition, we see that the variables that we used in section 2 (upto a prefactor of 2 since \( \kappa_\alpha = 2 \tau_\alpha \)) are given in terms of the Kähler coordinates as:

\[
\kappa_\alpha = \frac{2}{3} (T_\alpha - \bar{T}_\alpha) - \frac{i}{2(\tau - \bar{\tau})} \kappa_{\alpha\beta} (G - \bar{G})^\alpha (G - \bar{G})^\beta. \tag{3.5}
\]

Note the slightly confusing change of notation as compared to the previous section, which was introduced in order to follow the standard literature conventions. The fields \( \rho_\alpha \) in (3.4) are not to be mixed with the ones in the previous section. The coordinates \( \rho_\alpha \) discussed before actually correspond to the \( T_\alpha \)'s here with the difference that before the fields \( G^a \) were absent. Had we assumed that \( h_+^{(1,1)} = 0 \) the additional \( G^a \)-dependent term would vanish and everything would be the same, so we see that the results in literature are consistent with the neglect of the non-geometric moduli. However, if one really wants to stabilize all moduli in the generic case where \( h_+^{(1,1)} \sim h_-^{(1,1)} \sim O(100) \) we really need to use the coordinate basis given by (3.4). We will then describe in detail what happens in this case and show how all these moduli will be eventually stabilized in a manner similar to the KKLT and LVS procedures. In what follows we will separately discuss the resulting moduli potential and its stabilization for the tree-level case, for the case with added \( \alpha' \)-corrections and \( D3 \)-instantons. In the end we will be even able to draw conclusions on how the addition of worldsheet instanton corrections to the Kähler potential can influence the process of stabilization.

Note that once we derive the moduli potential from the Kähler metric and the basis of chiral fields \( \tau, T_\alpha, G^a \), we will be able to switch to the basis of real scalar fields using (3.4). It will turn out that minimization of the potential is easier in this new basis since the volume of the Calabi-Yau will depend only on the two-cycle moduli \( v^\alpha \) and not on the other scalars. Of course, once having stabilized all scalars one can always switch back to the initial chiral fields where the metric on the moduli space takes a simpler form.
3.1 Tree level

At tree level, once again the Kähler potential is

\[ K = -\ln\left[i \int_{\text{CY}} \Omega(z) \wedge \bar{\Omega}(\bar{z})\right] - \ln(-i(\tau - \bar{\tau})) - 2\ln\left(\frac{\kappa_\alpha v^\alpha}{6}\right), \tag{3.6} \]

where the \( \kappa_\alpha \)'s and \( v^\alpha \)'s have to be regarded as functions of the true Kähler coordinates \( (3.4) \). The superpotential is

\[ W = \int_{\text{CY}} \Omega(z) \wedge G_3 = \int_{\text{CY}} \Omega(z) \wedge (F_3 - \tau H_3) \equiv A(z)\tau + B(z), \tag{3.7} \]

where the \( z \)'s are the complex structure moduli. The full moduli potential can be calculated from Eq. (2.3). It is important here to stress that the potential at tree-level is positive semi-definite. This is not directly obvious from the expression, but is nevertheless true as it comes from the reduction of the \( \mathcal{N} = 2 \) to \( \mathcal{N} = 1 \) supergravity.

As found in \[42, 24\] the \( \mathcal{N} = 2 \) moduli potential is given by:

\[ V = \frac{18ie^\phi}{\kappa^2} \int_{\text{CY}} \frac{\Omega \wedge \bar{\Omega}}{\kappa} \left( \int_{\text{CY}} \bar{\Omega} \wedge G_3 \int_{\text{CY}} \bar{\Omega} \wedge G_3 + K_{kl} \int_{\text{CY}} \bar{\chi}_k \wedge G_3 \int_{\text{CY}} \bar{\chi}_l \wedge \bar{G}_3 \right), \tag{3.8} \]

where the \( \chi_k \)'s are a basis of harmonic \((2,1)\)-forms. Now this is manifestly positive semi-definite, implying the same property for the resulting \( \mathcal{N} = 1 \) potential. This means that any full minimum of the potential will be at \( V = 0 \) and local minima (if any) could be only de Sitter (at \( V > 0 \)).

With this information, we can now try to investigate the explicit form of the potential. The somewhat involved calculation of the Kähler metric and its inverse are carried out in Appendix B.2 and the results are in exact accordance with those in \[39, 45\]. There the results were summarized in the short formula

\[ K^{ij} K_i K_j = 4, \tag{3.9} \]

where the indices run over the Kähler moduli and the dilaton and not over the complex structure moduli. One can roughly break this sum into two contributions - a part in which the dilaton is involved plus a part coming only from the \( G^a \)'s and the \( T_\alpha \)'s as given by \( (B.34) \). This will be helpful when we want to search for minima of the moduli potential.

Using the tree-level superpotential \( (3.7) \) that does not depend on \( T_\alpha, G^a \) and the inverse Kähler metric \( (B.33) \) the moduli potential becomes:

\[ V = e^K \left( K^{z_a \bar{z}_b} D_{z_a} W D_{\bar{z}_b} \bar{W} + (A\tau + B)(A\bar{\tau} + B) \right) + \\
e^K \left( (\tau - \bar{\tau}) + \frac{3i(\tau - \bar{\tau})^2}{2\kappa} \kappa_{a\bar{a}b} v^a v^b \right) (|A|^2(\tau - \bar{\tau}) + \bar{A}B - A\bar{B}) \right). \tag{3.10} \]
As a self-consistency check one can easily see that in case when the non-geometric moduli vanish we go back to the usual tree-level potential (2.4) that is independent on the Kähler moduli.

Staring at this expression one can think of several ways to minimize it in directions of different moduli. One can start by calculating the partial derivatives for the different fields, setting them to zero and checking if the resulting equations could all be solved. This procedure could in principle lead to valid Minkowski or de Sitter vacua, but has to be checked explicitly for each specific choice of Calabi-Yau manifold and fluxes since the equations depend on $K^a z^a \bar{z}^b$ and $A, B$. Therefore we lose any predictive control of this outcome and can only go as far as noticing its possibility. However, there will be at least one minimum found, corresponding to the point where $D z^a W = 0$. We know this because the first term is actually positive semi-definite and only vanishes at the supersymmetric point, i.e. a full minimum of the potential can only be reached through fixing $D z^a W = 0$. This will result in a set of coupled equations for all complex structure moduli, in the end one would be able to express all $z^a$ in terms of the dilaton $\tau$ (as $W$ is a holomorphic function it does not depend on $\bar{\tau}$). Therefore the resulting potential will not have the first term of (3.10) and $A$ and $B$ will be unspecified functions of $\tau$. Their explicit dependence will again depend on the specific choice of manifold and fluxes. To minimize this new potential will again be only possible after looking case by case. Therefore we turn into the only case that is left, namely to stabilize all complex structure moduli and the dilaton to a supersymmetric minimum, $D z^a W = D \tau W = 0$. This indeed is the only possible case left since requiring $D T^\alpha W = 0$ results in overrestricting the fluxes as $W = 0$, and the other possibility $D G^a W = 0$ is already included in the above as we will show now. So, $D z^a W = D \tau W = 0$ results in stabilizing all complex structure moduli in terms of $\tau$ as before and then also fixing $\tau$ to be expressed in terms of the remaining free moduli:

$$\tau = -b \pm \sqrt{b^2 - 4c} \over 2,$$

with

$$b \equiv \left( \frac{3AB}{2} + \frac{A^2 \bar{B}}{2A} - \frac{9iA^2 \kappa_{a b} v^a b^a b^b}{\kappa} \right)$$

$$c \equiv \left( \frac{A B \bar{B}}{2A} + \frac{B^2}{2} - \frac{3iA^2 \kappa_{a b} v^a b^a b^b}{\kappa} \left( 2B + \frac{A \bar{B}}{A} \right) \right).$$

As $A$ and $B$ are set beforehand to be some functions of $\tau$, we see that the solution for $\tau$ cannot be simplified any further. We only know that it will eventually depend in some way through $b^a, v^a$ and can expect that this dependence is very slight and vanishing for large volumes (as $b$ and $c$ depend on those like $V_{CY}^{-2/3}$). The moduli potential in this simple case equals

44
\[ V = \frac{e^K e^{-2\phi} |W|^2}{4V_C^2} (\kappa_{\alpha a b} v^\alpha b^a b^b)^2. \]  

(3.12)

Now the potential is manifestly positive semi-definite once more. Clearly we can reach a full minimum \( V = 0 \) if and only if \( \kappa_{\alpha a b} v^\alpha b^a b^b = 0 \). In the initial Calabi-Yau \( \kappa_{\alpha \beta \gamma} v^\alpha \) has a signature \((1, h^{(1,1)} - 1)\). After the projection, \( \kappa_{\alpha \beta \gamma} v^\alpha \) is with signature \((1, h^{(1,1)}_+ - 1)\) and \( \kappa_{\alpha a b} v^\alpha \) with \((0, h^{(1,1)}_-)\). Then the only completely generic solution of \( \kappa_{\alpha a b} v^\alpha b^a b^b = 0 \) that is meaningful (i.e. we cannot have \( v^\alpha = 0 \) as the Calabi-Yau manifold will vanish) is to set \( b^a = 0 \) for all \( a \). This is the only possibility for a Minkowski vacuum in this case as clear also from Fig. 3.1. A dS vacuum might still be possible away from zero in this case but since we cannot know generically how \( \tau \) depends on the \( b^a \)’s we are again restricted to look at the problem case by case. Even finding dS minima in this case will not ensure that these are full minima of the starting potential \((3.10)\).

Therefore we are very restricted in terms of possible generic scenarios for stabilization of all moduli. Nevertheless, the case when \( \kappa_{\alpha a b} v^\alpha b^a b^b = 0 \) is genuinely a minimum of the potential, corresponding to vanishing of all terms dependent on the non-geometric Kähler moduli. Therefore we found a natural mechanism to stabilize all \( b^a = 0 \) to a supersymmetric value (since then \( D_G \alpha W = 0 \)). This mechanism leads us back to the no-scale potential that is stabilized to zero and is flat in the directions of the geometric Kähler moduli. We then go on to employ the KKLT and LVS procedures and calculate the effect of \( \alpha' \)- and instanton corrections on potential that now includes the non-geometric moduli.
3.2 $\alpha'$-corrections

Including $\alpha'$-corrections, the Kähler potential becomes

$$K = -\ln[i \int_{CY} \Omega(z) \wedge \bar{\Omega}(\bar{z})] - \ln(-i(\tau - \bar{\tau})) - 2\ln \left( \frac{\kappa v^a}{6} + \frac{\xi}{2} \left( \frac{\tau - \bar{\tau}}{2i} \right)^{3/2} \right),$$

(3.13)

with $\xi$ as defined before in (2.10). In this subsection we will stick to the tree-level superpotential (3.7) and therefore discuss only the effects of perturbative corrections to the process of moduli stabilization. Even only the addition of the $\alpha'$-corrections changes considerably the potential as we will see shortly. For now we just note that $V$ does not have to be positive semi-definite any more since $\xi$ could be either positive or negative depending on the the sign of the Euler number of the Calabi-Yau (c.f. Eq. (2.10)). As we will see the sign of $\xi$ will directly determine if $V$ is positive semi-definite or not.

To see this we calculate again the Kähler metric and its inverse in Appendix B.2. As expected, we obtain that Eq. (3.9) also holds in this case. The factor 4 is generic for a Kähler potential which is a logarithm of a homogeneous function of degree $-4$ as discussed in [40]. Therefore, the full potential now does not look too different from before:

$$V = e^K \left( K z^a z^b D z^a W D z^b \bar{W} + (A \tau + B)(A \bar{\tau} + \bar{B}) \right) - \alpha e^K \left( \frac{1}{(\tau - \bar{\tau})} + \frac{i}{4Y} \kappa_{ab} v^a b^b + \frac{3(\tau - \bar{\tau})^{1/2}}{(2i)^{3/2} Y} \right) (|A|^2 (\tau - \bar{\tau}) + AB - \bar{A}B),$$

(3.14)

with $a$ coming from the inverse metric as given by (B.43) and $Y = V_{CY} + \frac{\xi}{2} \left( \frac{\tau - \bar{\tau}}{2i} \right)^{3/2}$. To leading order in large Calabi-Yau volume, $a = - (\tau - \bar{\tau})^2$. Clearly, the minima of this potential are to be found in an analogous way to the minima of the tree-level case. Therefore the discussion following Eq. (3.10) holds once more and we will not repeat the same considerations again. The main point is that this expression is very hard to be minimized in a controlled way and this is why we again continue by going to the supersymmetric point in the directions of the complex moduli. Requiring $D z^a W = 0$ and $D \tau W = 0$ will stabilize all $z^a$'s and $\tau$ to being known functions of the Kähler moduli that depend on the Calabi-Yau manifold and are non-generic (c.f. (3.11)). In this case, the moduli potential becomes

$$V_{a'} = \frac{e^K e^{-2\phi}}{4 V_{CY}} \left( 3 \xi e^{\phi/2} + \frac{(\kappa_{ab} v^a b^b)^2}{V_{CY}^2} + O(V_{CY}^{-5/3}) \right).$$

(3.15)

Here, unlike in the tree level case, we do not have a general mechanism to prove that minimizing $V_{a'}$ will result in full minimization of (3.14) in all directions. Before $V = 0$ was an automatic full minimum, but this is no longer the case as the $\alpha'$-corrections
break this property. In this case we know that we have found minima only if it is at large volumes because of the argument used in the Large Volume Scenario after Eq. (3.16). In the case now

\[ V = e^K (K^{\alpha \beta} \tau^\alpha \bar{\tau}^\beta W D_{\tau} \bar{W} + K^{\tau \tau} D_{\tau} W D_{\bar{\tau}} \bar{W}) + O(V^{-2/3}) e^K (W D_{\tau} \bar{W} + \bar{W} D_{\bar{\tau}} W) + V_{\alpha'}. \]  

(3.16)

So it only makes sense to consider the minimization of \( V_{\alpha'} \) at large volumes, therefore we can consistently neglect the terms of order \( V^{-5/3} \) and higher in (3.15). We first observe that, as before, the term depending on the non-geometric moduli is positive definite and therefore it would be favorable if it vanishes: \( \kappa_{ab} v^a v^b = 0 \). If this is the case we are nevertheless left with volume dependence since the \( \xi \) term survives. Now we see how important the sign of \( \xi \) turns out to be:

- \( \xi > 0 \), i.e. \( \chi_{CY} < 0 \): The resulting potential is positive definite and vanishing as \( V_{CY} \to \infty \), i.e. this case is consistent with our assumptions but leads to decompactification of the Calabi-Yau. One can only hope that non-perturbative effects will eventually create a minimum at some finite large value of the volume (this is what happens in LVS).

- \( \xi < 0 \), i.e. \( \chi_{CY} > 0 \): In this case the minimum is when the volume goes to zero and the potential goes to \( -\infty \). Clearly none of these is in accordance with the approximations we have made so far, and we can only trust the result at large volumes where no minima can be found. Instanton corrections cannot help in generating large volume minima as we saw in details in section 2 in the discussion of the LVS. Therefore this case is undesirable and one needs very different approach in order to solve the problem of stabilizing the moduli for positive Euler number Calabi-Yau manifolds.

### 3.3 D-brane instanton corrections

Upto now we only considered the tree-level superpotential (3.7). Let us now see what happens if we compactify on a Calabi-Yau that meets the criteria to have nonzero D-instanton contributions. The superpotential is then:

\[ W = A(z) \tau + B(z) + \sum_{\alpha} A_{\alpha}(\tau, G) e^{-a_{\alpha} T_{\alpha}} = W_0 + W_{np}, \]  

(3.17)

where the sum over \( \alpha \) only goes through the cycles that admit D-instantons. Since this depends on the specific manifold, we shall assume for the moment that \( \alpha \) runs through all Kähler moduli. We will relax this condition soon because some of the elements of
the sum are very small and their presence is not relevant. The constants $A_{\alpha}$ in principle can depend on all other moduli except the $T_{\alpha}$’s but their explicit dependence is hard to determine and does not lead to further insight in the process of moduli stabilization at present (see e.g. section 2.4 of [32]).

$W_{np}$ now depends also on $G^a$ and $T_{\alpha}$ and therefore

$$D_{G^a} W = \sum_{\alpha} \frac{\partial A_{\alpha}}{\partial G^a} e^{-a_{\alpha} T_{\alpha}} + K_{G^a} W,$$

$$D_{T_{\alpha}} W = -a_{\alpha} A_{\alpha} e^{-a_{\alpha} T_{\alpha}} + K_{T_{\alpha}} W.$$  \hspace{1cm} (3.18)

We can directly consider that the Kähler potential includes $\alpha'$-corrections since we already showed that they will substantially change the minimization process and we cannot neglect them. Therefore, the full potential will become (in analogy to the LVS case Eq.(3.16)):

$$V = e^K \left( K^{\alpha \beta} D_{\alpha} W D_{\beta} \bar{W} + K^{\alpha \beta} D_{\alpha} W D_{\beta} \bar{W} \right) + O(V_{CY}^{-2/3}) e^K \left( W D_{\alpha} \bar{W} + \bar{W} D_{\alpha} W \right) + V_{\alpha'} + V_{np1} + V_{np2}. \hspace{1cm} (3.19)$$

Once again, at large volumes we can consistently set $D_{\alpha} W = D_{\alpha} \bar{W} = 0$. Given that the resulting equations have $G^a$ and $T_{\alpha}$ dependence that is suppressed at large volumes, we can safely assume that all complex structure moduli and the dilaton have been set to constants. Then,

$$V = V_{\alpha'} + V_{np1} + V_{np2}, \hspace{1cm} (3.20)$$

with

$$V_{\alpha'} = e^K \left( K^{ij} \partial_i W_{np} \partial_j \bar{W}_{np} \right) = e^K \left( \frac{e^{-\phi} |W|^2}{4 V_{CY}} \right) \left( 3 \xi e^{\phi/2} + \frac{(\kappa_{a\alpha b} k_{a\alpha b})^2}{V_{CY}} + O(V_{CY}^{-5/3}) \right), \hspace{1cm} (3.21)$$

$$V_{np1} = e^K \left( K^{ij} \partial_i W_{np} \partial_j \bar{W}_{np} \right) = e^K \left( -\frac{3}{2} e^{-\phi} \kappa_{Kab} \kappa_{abc} b^e + O(\kappa^0) \right) \sum_{\alpha, \beta} \partial_{G^a} A_{\alpha} \partial_{\bar{G}^b} \bar{A}_{\beta} e^{-a_{\alpha} T_{\alpha} + a_{\beta} T_{\beta}} + \hspace{1cm} (3.22)$$

$$+ e^K \left( -\frac{3}{2} (\kappa_{\alpha \beta} - 3/2 \kappa_{\alpha} \kappa_{\beta}) - \frac{3}{2} e^{-\phi} \kappa_{a\alpha b} \kappa_{abc} b^e \kappa_{bde} b^f + O(\kappa^0) \right) a_{\alpha} a_{\beta} A_{\alpha} \bar{A}_{\beta} e^{-a_{\alpha} T_{\alpha} + a_{\beta} T_{\beta}},$$

$$V_{np2} = e^K \left( K^{ij} \partial_i W_{np} \partial_j \bar{W}_{np} \right) = e^K \left( \frac{3}{2} \kappa_{K} + O(\kappa^{-2/3}) \right) \left( a_{\alpha} A_{\alpha} W e^{-a_{\alpha} T_{\alpha}} + a_{\alpha} \bar{A}_{\alpha} W e^{-a_{\alpha} T_{\alpha}} \right). \hspace{1cm} (3.23)$$

Let us for the moment try to discuss the minimization of (3.20) in the direction of the non-geometric moduli term by term. As before, $V_{\alpha'}$ is minimized if and only if all
$b^a = 0$. However, to minimize $V_{np1}$ and $V_{np2}$ is not so straightforward and we need to make some assumptions in order to proceed. The scaling of both expressions is being dominated by the exponential terms, and more precisely these are determined by the real part the term in the exponent, while the imaginary part decides on the sign. At large 4-cycle volumes the terms are very suppressed and we can safely ignore them as the exponential function drops to zero very rapidly. Therefore the dominating terms in $V_{np1}$ and $V_{np2}$ will be the ones corresponding to the smallest $\kappa_\alpha$, which we shall denote $\kappa_s$. Assuming for the moment that all other 4-cycles are too big, in the sense that $e^{-\kappa_\alpha} << e^{-\kappa_s}$ for all other $\alpha$, and using

$$T_s + \bar{T}_s = \frac{3}{2} \kappa_s + \frac{3i(\tau - \bar{\tau})}{4} \kappa_{sab} b^a b^b = \frac{3}{2} \kappa_s - \frac{3e^{-\phi}}{2} \kappa_{sab} b^a b^b,$$

we finally obtain

$$V = \frac{e^K}{V_{CY}} \left[ -\alpha(b) \ln(V_{CY}) e^{\frac{3}{4} a_s e^{-\phi} \kappa_{sab} b^a b^b} + \beta(b) + \gamma(b) \sqrt{\ln(V_{CY})} e^{\frac{3}{4} a_s e^{-\phi} \kappa_{sab} b^a b^b} \right],$$

(3.25)

where the exact dependence of $\alpha$ and $\gamma$ on the $b^a$'s is coming from (3.23) and (3.22):

$$\alpha(b) \simeq -2 \left( A_s \bar{W} e^{-i a_s (\frac{3}{4} \rho_s + \frac{3}{4} \kappa_{sab} (C_a b^a - c^a) b^a)} + \bar{A}_s W e^{i a_s (\frac{3}{4} \rho_s + \frac{3}{4} \kappa_{sab} (C_a b^a - c^a) b^a)} \right),$$

(3.26)

$$\beta(b) = \frac{e^{-2\phi} |W|^2}{4} \left( 3xe^{\phi/2} + \frac{(\kappa_{sab} b^a b^b)^2}{V_{CY}} \right),$$

(3.27)

$$\gamma(b) \simeq 6 \left( \sqrt{\frac{3}{a_s}} - \frac{3e^{-\phi}}{2\sqrt{\ln(V_{CY})}} \kappa_{sab} b^a \kappa_{sbc} b^c \right) a_s^2 |A_s|^2 - \frac{2e^{-\phi}}{3\sqrt{\ln(V_{CY})}} \kappa_{ab} \partial_{[C_a} A_s \partial_{C_b]} \bar{A}_s - \frac{ie^{-\phi}}{\sqrt{\ln(V_{CY})}} \kappa_{sab} b^a a_s \left( A_s \partial_{C_a} \bar{A}_s - \bar{A}_s \partial_{C_a} A_s \right).$$

(3.28)

Here we already made the basic assumption of the Large Volume Scenario that the small cycle volume scales as $e^{-\frac{3}{4} a_s \kappa_s} \approx V_{CY}^{-\frac{1}{4}}$, which is dictated by the minimization of the potential in the direction $\kappa_s$.

In (3.26) for the first time we explicitly see some dependence on the moduli $\rho_s, c^a$ defined through (3.1), (3.3), and (3.4). This means we are allowed to stabilize them in a way that will minimize $\alpha$. Since they appear only in the imaginary part of the exponent they can only determine the sign of $\alpha$ but not its magnitude (they can give a relative prefactor between -1 and 1). Therefore it is clear that $\rho_s, c^a$ arrange themselves in a way to make the expression as negative as possible (i.e. the prefactor would be -1). Since they appear in the term $a_s A_s \bar{W} e^{-i a_s (\frac{3}{4} \rho_s + \frac{3}{4} \kappa_{sab} (C_a b^a - c^a) b^a)} + c.c.,$ there will be one equation to constrain the possible values of $\rho_s$ and the $c^a$'s. This will be enough to stabilize $\rho_s$ as in the original LVS and the $c^a$'s still remain unstabilized. Therefore
\(\alpha > 0\) with certainty.

On the other hand, we know that \(\gamma(b)\) must be positive as it comes from the inner product of the vector \(\partial_i W\) with itself. \(\beta\) is also positive by assumption since a negative value will not lead to consistent minima as shown in the previous subsection. Then, for the full potential to be as small as possible the remaining free moduli will try to make the magnitude of \(\beta\) and \(\gamma\) terms as small as possible and the magnitude of \(\alpha\) term as big as possible.

To decrease the exponent after \(\gamma(b)\), \(\kappa_{sab} b^a b^b\) needs to be large negative, while increasing the magnitude of the exponent after \(\alpha(b)\) can be done by making \(\kappa_{sab} b^a b^b\) large positive, thus leading to a conflict between those two terms. On the other hand making \(|\kappa_{sab} b^a b^b|\) large is also not a good choice since we will increase \(\beta(b)\) too much. Therefore we see that the value at which we had stabilized the non-geometric moduli at tree-level, i.e. \(b^a = 0\), is still an admissible value at first sight.

If all \(b^a = 0\) then we can largely simplify the expressions for \(\alpha(b)\), \(\beta(b)\) and \(\gamma(b)\) and will make the exponents trivial. In fact, it is easy to check that we get back the Large Volume Scenario, \(\alpha(b) = \alpha_{LVS}\), \(\beta(b) = \beta_{LVS}\), and \(\gamma(b) = \gamma_{LVS}\). In particular Eq.(2.17) is reproduced and all the discussion that follows it about the minimization of the potential holds once more. Therefore we see that large volume minima can still exist under the same conditions as before, i.e. some particular relative weight of the prefactors \(\alpha, \beta, \) and \(\gamma\) (c.f. (2.17)).

Let us turn again to the stabilization of \(b^a\)'s and check how stable they are for small variations around the zero now that we know generically how the potential behaves also in the geometric moduli directions (by the LVS scenario). A slight move of \(b^a\) away from zero will result in a positive uplift of \(\beta\) which will be suppressed at large volumes since it is of lower power. In the same time, it will change the relative weight of the exponents after \(\alpha\) and \(\gamma\) (which is directly relevant for the LVS). Such a change will therefore shift the large volume minimum in the geometric moduli directions, and the resulting value of the potential at the new minimum will be different. Formally, at this point we can check our intuition by using the LVS potential with the extra \(b^a\)-dependence, for the moment neglecting the \(b^a\)-dependence of \(\alpha, \beta, \) and \(\gamma\) as it is suppressed compared to the dependence in the exponents. We will later justify this assumption further. Then,

\[
\frac{\partial V}{\partial b^a} = \frac{e^K}{V_{CY}} \kappa_{sab} b^c e^\frac{3}{4} a_s e^{-\frac{\kappa}{\kappa_{sab}} b^a b^b} \sqrt{\ln(V_{CY})} \left[ -\frac{3}{2} \alpha \sqrt{\ln(V_{CY})} + 3 \gamma e^\frac{3}{4} a_s e^{-\frac{\kappa}{\kappa_{sab}} b^a b^b} \right].
\]

We have extrema of the potential in all non-geometric directions whenever \(\frac{\partial V}{\partial b^a} = 0\) for all \(b^a\). Clearly this is satisfied by \(b^a = 0\) for all \(a\), while other solutions can be found only for specific cases depending on the form of the intersection numbers \(\kappa_{sab}\).

In the former case our assumption that \(\alpha, \beta, \) and \(\gamma\) do not depend on the \(b^a\)'s is clearly...
satisfied. We can go further and compute the matrix of second derivatives:
\[
\left( \frac{\partial^2 V}{\partial b^a \partial b^b} \right)_{b^b=0} = 3e^K \kappa_{sab} \frac{1}{V_{CY}} \left( -\frac{1}{2} \alpha \ln(V_{CY}) + \gamma \sqrt{\ln(V_{CY})} \right) .
\] (3.30)

In typical cases \(\alpha\) and \(\gamma\) are of the same order of magnitude so the term in brackets is negative. On the other hand, the matrix \(\kappa_{sab}\) could in some cases be negative definite since it comes from the orientifold projection from a Calabi-Yau manifold. We know from before that \(\kappa_{sab} v^a\) is negative definite. Since \(v^a\) is positive for all \(a\), it is plausible that the signature of \(\kappa_{sab}\) as a subpart of \(\kappa_{sab} v^a\) is also \((0, 1)\). In this case, \(\left( \frac{\partial^2 V}{\partial b^a \partial b^b} \right)_{b^b=0}\) is positive definite and therefore \(b^a = 0\) is a full minimum of the potential with the large volume scenario holding for "good" values of \(\alpha, \beta, \gamma\). This argument is however not valid when \(\gamma >> \alpha\), as in this case the term in the brackets becomes positive and \(b^a = 0\) is a maximum instead of a minimum (it will be a minimum only if \(\kappa_{sab}\) is positive definite, which is highly unlikely). So the possibilities for a large volume minimum which we obtain in a controlled way are restricted to the case when \(\beta >> \alpha \approx \gamma\).

We can nevertheless show the existence of another class of minima, for which explicit solutions cannot be given. From (3.35) we see that are extrema also at
\[
\frac{2}{\alpha^2} a_s e^{-\phi} \kappa_{sab} b^a b^b = \ln \left( \frac{2\sqrt{\ln(V_{CY})}}{2\gamma} \right).
\] (3.31)

This second possibility for extrema is genuinely new as it holds for all cases where \(\kappa_{sab}\) is not negative definite as we require \(\kappa_{sab} b^a b^b > 0\) (<0 only when \(\gamma >> \alpha\) but we will here focus on the usual case when \(\alpha \sim \gamma\)). This equation can always be satisfied if \(\kappa_{sab}\) has at least one positive eigenvalue and this will lead to a constraint of one of the \(b^a\)'s in terms of the others, i.e. we have extrema for a \(h_{1,1} - 1\) dimensional hypersurface parameterizing the space of \(b^a\) moduli. On this hypersurface, the matrix of second derivatives reads
\[
\left( \frac{\partial^2 V}{\partial b^a \partial b^b} \right)_{hypersurface} = \frac{9e^K e^{-2\phi} a_s^2 \alpha^2 (\ln(V_{CY}))^{3/2}}{4\gamma V_{CY}^{3/2}} \kappa_{sac} b^c \kappa_{sbd} b^d ,
\] (3.31)

i.e. we are at minimum of the potential as long as \(\kappa_{sac} b^c \kappa_{sbd} b^d\) is positive definite. Our claim is now that this is always satisfied away from the hypersurface \(\frac{2}{\alpha^2} a_s e^{-\phi} \kappa_{sab} b^a b^b = \ln \left( \frac{2\sqrt{\ln(V_{CY})}}{2\gamma} \right)\). This is the case because for a contraction with an arbitrary vector \(b^a\), \(b^a \kappa_{sac} b^c \kappa_{sbd} b^d b^b = (\kappa_{sab} b^a b^b)^2\) which is positive unless \(b^a\) is orthogonal to \(\kappa_{sab} b^b\). This means that, as expected, for movement along the hypersurface the potential remains the same, while away from it the potential rises. The situation is reminiscent of the "Mexican hat" potential of the Higgs mechanism, where the hypersurface of constant potential is a circle in the two-dimensional plane (c.f. Fig. (3.2)).

As opposed to this circle, in our case the manifold of constant potential is of a priori unknown dimension and needs not be compact (it is compact only if \(\kappa_{sab}\) has
Figure 3.2: The famous "Mexican hat" potential of the Higgs mechanism. In our case this is exactly what we find if $\kappa_{sab}$ is positive definite.

Figure 3.3: A generalized "Mexican hat" potential where we allow $\kappa_{sab}$ to have both positive and negative eigenvalues. Here the surface of constant minimal potential is not bounded and extends to infinity.
only positive or zero eigenvalues), an example of this depicted on Fig. 3.3.

Let us now turn back to the lower-order volume corrections in $\beta(b)$ and the full expressions for $\alpha(b)$ and $\gamma(b)$. From the full $\beta(b)$ plugged in (3.25) it is clear that the correction breaks one symmetry of the potential - before it was just a function of $V_{CY}$ and $\kappa_{sac}b^a b^b$, while now the full dependence on $\kappa_{aabc}b^a b^b b^b$ is exhibited. As the new term is subleading in volume it will create a small perturbation of the existing potential that was described above. The new term will be again of the shape of Fig. 3.1 but will no longer be dominant. For $\kappa_{sab}$ negative definite this will not change the minima given by $b^a = 0$ for all $a$. For this case also $\alpha$ and $\gamma$ are unchanged. In the other case, the new term will generate some additional gradient over the hypersurface of constant potential, such that the minimum will now be still at points of the manifold that will additionally minimize $\kappa_{sabc}b^a b^b b^b$, i.e. at the points on this surface where the $b^a$’s are closest to zero as the other points will get a higher uplift. It is important to stress that in the large volume approximation this extra term cannot generate potential to entirely wipe out the minima on the manifold, but will simply result in small rise of the potential in some or all of the directions along the hypersurface. The potential corrections to $\alpha$ and $\gamma$ from the nonzero value of $\kappa_{sabc}b^a b^b$ cannot influence the process either. $\alpha$ will not be changed at all as any changes of $b^a$’s result in change of the numerical value of the $\rho_s$ that will make sure $\alpha$ remains maximal. The magnitude of $\gamma$ is less clear but it will tend to decrease and thus it is not of crucial importance for the overall minimum as explained in the previous section. It is hard to give a more precise statement since the problem gets out of analytical control. Nevertheless it is not hard to imagine that relatively small values such as $\kappa_{sabc}b^a b^b b^b = \frac{4}{3\rho_s} e^b \ln \left( \frac{\alpha V_{CY}^\gamma}{2} \right)$ will not lead to major changes in the form of the minima due to change of $\alpha$, $\beta$, and $\gamma$, though it can also create some non-flat directions along the hypersurface of previously constant potential. Together with the additional lower order correction this will mean that some more of the $b^a$ moduli will be stabilized to a constant value and will gain mass. The mass generated this way will naturally be lighter than the masses for the previously stabilized fields due to the fact that it comes from a subleading term. This might potentially have some phenomenological implications as we will discuss in the next section.

In order to show that a full minimum exists, we need to consider also the mixed directions in $V_{CY}$ and $b^a$. In the case that all $b^a$’s are zero this is fairly simple since $\frac{\partial^2 V}{\partial b^a \partial V_{CY}} = 0$ and therefore we can minimize separately the potential in the volume direction, leading to the ordinary Large Volume Scenario as in this case all new terms in the potential coming from the $b$ fields vanish. The case where we have a hypersurface
of solutions is much more subtle. Here,

\[ \left( \frac{\partial^2 V}{\partial b^a \partial V_{CY}} \right)_{\text{hypersurface}} = \frac{3e^K e^{-\phi} a_s a^2 (\ln(V_{CY}))^{3/2}}{4\gamma V_{CY}^2} \left( \frac{\ln(V_{CY}) - 1}{\ln(V_{CY})} \right) \kappa_{sab} b^b \]

and therefore one cannot just minimize the potential in the volume direction and in the \( b \)-fields independently as one would hope. Nevertheless, we know there will always be at least one minimum due to the following considerations. Let us consider for a moment again the potential as a function of two variables: \( V_{CY} \) and \( \kappa_{sac} b^a b^b \). We know how the function behaves for sure at three special values of \( \kappa_{sac} b^a b^b \): at zero and positive and negative infinity. At \( -\infty \) the terms proportional to \( \alpha \) and \( \gamma \) vanish and we are left with the \( \beta \) term corrected by a (possibly infinite) subleading positive term - in this case the potential in the volume direction starts from \( +\infty \) at small \( V_{CY} \) and goes steadily to zero with increasing volume without changing sign. At \( \kappa_{sac} b^a b^b = 0 \) we reproduce the LVS, therefore the potential starts at positive infinity for small volumes, goes below zero and has a minimum at a large volume, and goes to zero from below as \( V_{CY} \to \infty \). For \( \kappa_{sac} b^a b^b \to \infty \), the potential goes exponentially to positive infinity since the \( \gamma \) term dominates, thus for any value of the volume the potential remains positive. From this partial picture of the potential dependence one can clearly see that the potential must have a minimum for some finite values of both \( V_{CY} \) and \( \kappa_{sac} b^a b^b \). As discussed above after stabilizing \( \kappa_{sac} b^a b^b \) the low order corrections will tend to stabilize separately the \( b^a \)'s that remain free. We can further see how the value of the volume at the minimum will change generally. Compared to the LVS, plugging \( \frac{3}{4} a_s e^{\phi} \kappa_{sab} b^a b^b \) in (3.25) we obtain:

\[ V = \frac{e^K}{V_{CY}} \left[ \frac{-\alpha^2}{4\gamma} (\ln(V_{CY}))^{3/2} + \frac{e^{-2\phi}|W|^2}{4V_{CY}} \right] \kappa_{sab} b^b \]

This clearly leads to lower volume minima because of the higher power of \( \ln(V_{CY}) \) in the \( \alpha \) term. Observe that, amazingly, this preserves the possibility of large volume minima since the constants \( \alpha \) and \( \gamma \) combine in this specific form. In the LVS there were two possibilities for large volume minima, when \( \beta >> \alpha \sim \gamma \), or \( \gamma >> \alpha \sim \beta \). Now both these cases will lead to \( \frac{\alpha^2}{\gamma} << \beta \), still preserving the large volume minima. Thus the minima will still be at large enough volumes, even though not as large as the ones predicted from the standard LVS (due to the \( (\ln(V_{CY}))^{3/2} \) instead of \( \ln(V_{CY}) \)).

Therefore, we emerge with two main scenarios for stabilization of the non-geometric moduli that entirely depend on the specific Calabi-Yau intersection numbers:

- \( \kappa_{sab} \) negative definite: In this case \( b^a = 0 \) for all \( a = 1, ..., h^{1,1}_n \) and the standard LVS is reproduced.
• $\kappa_{sab}$ having at least one positive eigenvalue: Here explicit minima cannot be found analytically, but it is clear that at least one of the $b^a$’s will be stabilized to a non-zero value and that the minimum will be at smaller (but still large) volume for suitable choice of parameters that hold for the LVS.

In both cases there might be additional complication if $\kappa_{sab}$ has also zero eigenvalues. In this case there will be flat directions at leading order, but these will be fixed by the subleading terms and therefore we will not worry about this possibility (we will see soon that for more small moduli this case is highly unlikely).

Generalizing these conclusions for many small moduli is straightforward with the only subtlety coming from $V_{np1}$ since there we obtain a mix of exponential terms also for different cycles. Any small four-cycle will lead to a corresponding non-perturbative contribution to the $\alpha$ and $\gamma$ terms:

$$V = \frac{e^K}{V_{CY}} \ln(b) + \left( -\alpha_1(b) \ln(V_{CY}) e^{\frac{3}{2}a_1 e^{-\phi_{s1}} b^b} + \gamma_1(b) \sqrt{\ln(V_{CY})} e^{\frac{3}{2}a_1 e^{-\phi_{s1}} b^b} \right) + \ldots + \left( -\alpha_n(b) \ln(V_{CY}) e^{\frac{3}{2}a_n e^{-\phi_{s1}} b^b} + \gamma_n(b) \sqrt{\ln(V_{CY})} e^{\frac{3}{2}a_n e^{-\phi_{s1}} b^b} \right) -$$

$$-6e^K \sum_{i\neq j} \left[ i e^{-\phi_{ab}} \kappa_{s_{ij}bc} e^{\frac{3}{2}a_{s_{ij}} e^{-\phi_{s_{ij}}} b^b} \left( a_{s_{ij}} A_{s_{ij}} \partial G e A_{s_{ij}} + a_{s_{ij}} A_{s_{ij}} \partial G e A_{s_{ij}} \right) e^{-(a_{s_{ij}} T_{s_{ij}} + a_{s_{ij}} T_{s_{ij}})} + \right] + \left( \frac{3}{2} \kappa_{s_{ij}} + \frac{3}{2} e^{-\phi_{ab}} \kappa_{s_{ij},ac} e^{\frac{3}{2}a_{s_{ij}} e^{-\phi_{s_{ij}}} b^b} \right) a_{s_{ij}} a_{s_{ij}} A_{s_{ij}} A_{s_{ij}} e^{-(a_{s_{ij}} T_{s_{ij}} + a_{s_{ij}} T_{s_{ij}})}, \quad (3.34)$$

where the constants $\alpha_i$ and $\gamma_i$ are defined in analogy to (3.26 - 3.28) with addition of the index $i$ where needed to distinguish between different small volume cycles. Thus we can stabilize all $\rho_{si}$ by maximizing each $\alpha_{si}$ separately. Here we see that the second part of (3.34) is a brand new term that mixes in a complicated way all moduli $\kappa_{s_{ij}}, \rho_{si}, b^a, c^a$. Its value is ultimately restrained by the condition $V_{np1} \geq 0$ so it must be smaller than the $\gamma_i$ contributions. As these contributions themselves were shown to be unimportant when $\alpha \sim \gamma$ we shall ignore the whole second part of the formula for the potential, just noting that it will solve the problem with the unstabilized $c^a$’s as it exhibits a nontrivial dependence on them. We then try once again to minimize the remaining expression with respect to the $b^a$-moduli:

$$\frac{\partial V}{\partial b^a} = e^K \sqrt{\ln(V_{CY})} \left[ \kappa_{s_{ab}} b^b e^{\frac{3}{2}a_{s_{ab}} e^{-\phi_{s_{ab}}} b^b} \left( -\frac{3}{2} \alpha_1 \sqrt{\ln(V_{CY})} + 3 \gamma_1 e^{\frac{3}{2}a_1 e^{-\phi_{s1}} b^b} \right) + \ldots + \kappa_{s_{ab}} b^b e^{\frac{3}{2}a_{s_{ab}} e^{-\phi_{s_{ab}}} b^b} \left( -\frac{3}{2} \alpha_n \sqrt{\ln(V_{CY})} + 3 \gamma_n e^{\frac{3}{2}a_n e^{-\phi_{s1}} b^b} \right) \right] \quad (3.35)$$

The extrema will be given similarly to above, either by $b^a = 0$ for all $a$ or $\frac{3}{4} a_{s_{ab}} e^{-\phi_{s1}} b^b = \ln \left( \frac{a_{s_{ab}} \sqrt{\ln(V_{CY})}}{2 \gamma a} \right)$, ..., $\frac{3}{4} a_{s_{ab}} e^{-\phi_{s_{ab}}} b^b = \ln \left( \frac{a_{s_{ab}} \sqrt{\ln(V_{CY})}}{2 \gamma a} \right)$ for $n$ small moduli (usually $n = h_{+1}^1 - 1$). Of course, in principle one can also find extrema by equating to zero the whole expression above, but this is impossible to solve analytically and one loses control over
the type of extrema. Note that \( \alpha_1 \simeq \alpha_2 \ldots \simeq \alpha_n \) and \( \gamma_1 \simeq \gamma_2 \ldots \simeq \gamma_n \) as they differ only by the small difference in the proportionality constants of the four-cycles with respect to the logarithm of volume. The classification of these "controlled" extrema is however much more involved and depends on all \( \kappa_{s_{1,ab}}, \ldots, \kappa_{s_{n,ab}} \). In general both cases are possible to realize since now the condition for \( b^a = 0 \) to be minimum is relaxed and just states that the combined matrix as sum of sub-matrices should be negative definite. In the same time the second extremum is always minimum as long as it can be realized, i.e. as long as for each small cycle \( s_i, \kappa_{s_{i,ab}} b^a b^b > 0 \) is equation with a solution (i.e. \( \kappa_{s_{i,ab}} \) has at least one positive eigenvalue). Therefore, in the case of many small four-cycles, both types of minima are possible to be realized within a specific Calabi-Yau manifold. Here the number of small moduli \( h^{1,1}_+ - 1 \) and the number of \( b \)-fields \( h^{-1}_- \) are important. If \( h^{1,1}_+ - 1 > h^{-1}_- \) we will have too many constraints and the only hope for obtaining minima is at \( b^a = 0 \). On the contrary, if \( h^{1,1}_+ - 1 < h^{-1}_- \) some \( b^a \)'s can remain free at least until lower-order corrections to the potential are introduced. Note that for the case \( \gamma \gg \alpha \) the above considerations hold with reverse sign, i.e. the first minima need an overall positive definite matrix, while the second type of minima are practically always possible since naturally all \( \kappa_{s_{i,ab}} \) will have a negative eigenvalue (remember that \( \kappa_{a'b'} v^a \) is negative definite). This case (\( \gamma \gg \alpha \)) is more dangerous to discuss since the approximation made above of neglecting the whole second part of (3.34) might be not justified and decrease substantially the effective value of \( \gamma \). This is not a problem when \( \alpha \sim \gamma \) since one can obtain large volume minima even at lower \( \gamma \) values in this case. The minimization of this term, together with the minimization of \( V_{mp2} \) with respect to the \( \rho_\alpha, c^a \) will eventually ensure that all of them are stabilized (again, unless \( h^{1,1}_+ < h^{-1}_- \)).

We have thus found possible large volume minima not only for vanishing \( b^a \)'s, but also for non-zero values. As we will see in the next subsection these minima could be destabilized by other instanton effects, so the minimization of the non-geometric moduli turns out to be important step in the process and cannot be just neglected. Therefore we believe that the neglect of the non-geometric moduli in common literature is not very well justified and that the topic deserves greater attention.
3.4 Worldsheet instanton corrections

Another correction to the Kähler potential is given by worldsheet instantons as discussed in detail in [32] and [33]. Following the notation in [32],

\[ K = -\ln(-i(\tau - \bar{\tau})) - 2\ln \left( V_{CY} + \left( \frac{\tau - \bar{\tau}}{2i} \right)^{3/2} \left( \frac{\xi}{2} - 4\Im F(\tau, G) \right) \right), \]

(3.36)

with

\[ \Im F = \sum_{\beta \in H_2^+} \sum_{n=1}^{\infty} \frac{n_0^2}{n^3} \cos \left( n \frac{k^\beta_a(G - \bar{G})^a}{\tau - \bar{\tau}} \right), \]

(3.37)

where \( k^\beta_a = \int^\beta \omega_a \). The numbers \( n_0^\beta \) are the topological invariants of Gopakumar-Vafa [44], associated with each element of the homology. This new Kähler potential includes infinite sum over \( n \) and another sum over the elements of the homology \( H_2^+ \) of the CY manifold. This makes the metric very hard to invert, i.e. one can only invert it after knowing the dimensions of the homology. Therefore we cannot present a generic inverse of \( K_{ij} \) that is manifold independent as was the case before. However, we can use the intuition from previous results to draw quite generic conclusion on how worldsheet instantons can influence the moduli stabilization. Note that the corrections are subleading in volume, i.e. as far as Kähler moduli are concerned,

\[ K = -2\ln(V_{CY}) - 2 \left( \frac{\tau - \bar{\tau}}{2i} \right)^{3/2} \left( \frac{\xi}{2} - 4\Im F(\tau, G) \right) + O(V_{CY}^{-2}). \]

(3.38)

Therefore the worldsheet instantons will appear in the end result the same way as the \( \alpha' \)-corrections, i.e. in the definition of the \( \beta \) term in the LVS.

If we consider more closely the dependence of \( \Im F \), we see that:

\[ \frac{\partial \Im F}{\partial \tau} = -\sum_{\beta \in H_2^+} \sum_{n=1}^{\infty} \frac{n_0^2}{n^3} \cos \left( n \frac{k^\beta_a(G - \bar{G})^a}{\tau - \bar{\tau}} \right) \]

\[ \frac{\partial \Im F}{\partial G^a} = \sum_{\beta \in H_2^+} \sum_{n=1}^{\infty} \frac{n_0^2}{n^3} \cos \left( n \frac{k^\beta_a(G - \bar{G})^a}{\tau - \bar{\tau}} \right). \]

(3.39)

For \( \frac{(G - \bar{G})^a}{\tau - \bar{\tau}} = b^a = 0 \) the first term vanishes and one is left only with contribution to \( K_{G^a} \). However, if we follow closely the computations in Appendix B.2 for obtaining Eq.(B.45) we will see that no contribution to it comes from \( K_{G^a\bar{G}^b}, K_{G^a\bar{T}^a}, K_{G^a\bar{T}^b} \). This means that contributions from the partial derivative \( \frac{\partial \Im F}{\partial G^a} \) will appear only in very subleading terms of order \( V_{CY}^{-2} \). Therefore we can effectively consider \( \Im F \) constant for the case when we stabilize the non-geometric moduli to \( b^a = 0 \) as suggested by
all previous discussion. Note that this further makes sense in this case because it maximizes \( \text{Im} \mathcal{F} \) at \( \frac{(G-ar{G})^a}{\tau-ar{\tau}} = 0 \) for all \( a \) we get:

\[
\text{Im} \mathcal{F} = \sum_{\beta \in \mathcal{H}_2} \sum_{n=1}^{\infty} \frac{n^0}{n^3} = \zeta(3) \sum_{\beta \in \mathcal{H}_2} n^0_{\beta},
\]

since \( \cos 0 = 1 \). The maximizing of \( \text{Im} \mathcal{F} \) means minimization of the full potential because this term appears with a minus sign in front, i.e. it decreases the value of the (previously) positive \( \beta \) term:

\[
\beta_{\text{new}} = -\frac{3e^K e^{-3\phi/2} |W|^2 \zeta(3)}{2} \left( \frac{\chi}{4(2\pi)^3} + \sum_{\beta \in \mathcal{H}_2} n^0_{\beta} \right).
\]

(3.41)

Therefore generically \( \beta \) tends to decrease by worldsheet instantons. If it decreases so much that it becomes negative we will no longer have any minima in the volume direction as discussed in section 2 for the LVS. On the other hand, if \( \beta \) becomes of the order of \( \alpha, \gamma \) we will only have small volume minima. Therefore one can only hope that the Calabi-Yau orientifold does not allow for larger values of the worldsheet instanton correction as this can spoil the whole process of moduli stabilization. A large enough \( \gamma \) will still enable the desired minima for small positive \( \beta \), but as explained in the previous subsection this no longer complies with stabilizing the non-geometric moduli to \( b^a = 0 \).

Note that the above discussion is fact quite general and does not necessarily have to hold only for all \( b^a = 0 \), although this is the only case that can be handled with certainty. For the non-zero values the \( b \)-fields one can still argue that they will still tend to take such values as to make the \( \beta_{\text{new}} \)-term as small as possible. Therefore the minima of the potential away from the point \( b^a = 0 \) will shift in order to make the potential smaller, which in turn will mean that the volume will have to be stabilized at a lower value as compared to before. As we see the risk of destroying the Large Volume Scenario by adding worldsheet instantons is quite generic and one needs further investigation in order to make sure phenomenologically accepted vacua are still present.
4

Discussion and Conclusion

We made one more step in the process of full stabilization of the scalar fields in the compactification of type II B string theory on Calabi-Yau three-folds. As seen, the search for supersymmetric and non-supersymmetric minima of the moduli potential is a nontrivial task and one often needs numerical simulations to verify the intuitive expectations. Furthermore, many approximations and simplifications are employed in the process and it is not always granted that these are justified in all possible models. Clearly, perturbative and non-perturbative corrections play important role and it is unfortunate that at present there is no full classification of possible terms that can appear in the Kähler potential and the superpotential.

Nevertheless, the present work sheds some light on the stabilization of non-geometric moduli that can potentially help for further progress in the field. We will now try to briefly describe some possibilities that arise from the consideration of these additional moduli and then summarize the open questions and directions in the moduli stabilization process.

4.1 Phenomenological implications

The non-geometric moduli discussed up to now are also known in literature by the name of axions [47, 48, 49]. In type II B they arise from the 2-form fields $B_2$ and $C_2$ and the 4-form field $C_4$ as given by Eqs. (3.1) and (3.3). There are a few scenarios which employ these axions for phenomenological purposes.

One scenario proposes that axions provide a solution to the CP problem in QCD, i.e. they appear as an extension of the Standard Model. The idea, developed initially by [50], is to have a dynamical field (the axion) that couples weakly with the strong
force and thus generates the well-known experimentally CP violation. The axion is then required to be stabilized and have a certain mass scale. The resolution of the CP problem which is in accordance with other cosmological observations constrains the mass scale of the axion in a certain fairly small range of values. For more specific details one can check [48].

It is therefore natural to wonder if the axions that we considered in the previous section can provide phenomenologically interesting examples. Up to now people studied the axions coming from the $C_4$-field and there are no-go theorems [48] stating that they are far too heavy to be of any interest. However, the axions from the 2-forms were not considered as they come from the $h^{1,1}$ cohomology after the orientifolding. We saw in section 3.3 that some of these axions can be stabilized at a lighter mass compared to the others, depending on the choice of manifold. Thus, they can possibly provide the "missing" axion for QCD.

Another possibility to use axions is in cosmology, where one can imagine that the early stages of the universe are governed by a large number of axions that cause inflation [51] (also called N-flation). For this purpose one again requires the axions to be light enough. In [49] the N-flation scenario with all axions coming from type II B was considered and made plausible in some specific toy models. Therefore our work further extends the possibility to study this idea as it provides a realistic model for axion stabilization.

Other inflationary scenarios build on stabilizing moduli have been also proposed. For example, [52] considers the possibility of some of the Kähler moduli to be driving inflation. A more involved possibility of D-brane inflation involving the effects of moduli stabilization is discussed in [53]. Generally, there are many inflation models and correspondingly many scenarios to embed inflation within string theory, but unfortunately these are still far away from being testable experimentally so all string theorists can do at present is to come up with different possibilities.

4.2 Further developments

The topic of moduli stabilization attracts more and more scientists as it seems to be an inevitable step between string theory and experiments. Therefore a lot of research has been and is presently conducted on all possible aspects of moduli stabilization discussed or mentioned in this paper. Much attention has been drawn on the conjectured mirror symmetry between type II A and type II B flux vacua [54]. This symmetry can help for better understanding of perturbative and non-perturbative corrections to the
moduli potential as it is enough to compute them on one side (the type II A or the type II B) in order to obtain the corresponding behavior on the other side.

Unfortunately, explicit compactifications of type II A are hard to realize due to the highly warped geometry that one gets out of the initial Calabi-Yau manifold. [55] make some first steps towards generalizing the basic results from the Calabi-Yau moduli spaces to the warped geometry moduli spaces. Also some non-perturbative effects have been calculated [56]. A simple model of fixing all moduli with flux compactification has been discussed in [57]. Further research is needed if one wants to establish with certainty the precise relations between type II A and type II B flux compactifications, but the general hope is that this is only a matter of time and mathematical development.

In the same time working models for type II B flux compactifications are being improved. There are various extensions and upgrades of the Large Volume Scenario including different ways for supersymmetry breaking and various phenomenological discussions (check [58] and references therein). Also different possible ways for uplifting the resulting AdS vacuum to dS have been discussed (e.g. via including also F-terms [59]), but no conclusion has been reached yet.

An important issue for all stabilization scenarios is the discussion of quantum corrections and their regime of importance, i.e. how suppressed they are with volume. References [28] and [60] study this topic in detail and show that string loop corrections for "Swiss-cheese" CY manifolds are subleading compared to the \(\alpha'\)-corrections, but there may be other types of manifolds for which this is not satisfied. Other possible \(\alpha'\)-corrections are known to be less important compared to the ones discussed in the LVS, i.e. it seems that the LVS is safe from further \(\alpha'\) and \(g_s\) corrections. However, there might be other corrections form DBI actions and \(\mathcal{N} = 1\) supergravity that are of importance. If we are to claim that realistic string compactifications have been found, much better understanding of all these corrections are needed. Needless to say, same holds for instanton corrections that appear both in \(W\) and \(K\) - as seen in the previous section world-sheet instantons have the potential to fully break down the LVS and one can imagine other instanton corrections can also have an important influence (hopefully in the opposite direction).

It is therefore also clear that the full classification of flux vacua goes necessarily through classification of the various types and numbers of Calabi-Yau manifolds that can appear. Clearly one would be satisfied if moduli can be generically stabilized for any particular choice of underlying manifold, but this does not seem to be the case at
present. As seen in Appendix A, even the very simple 2-moduli examples of complete intersection Calabi-Yau’s do not have to be of "Swiss-cheese" type and have a very complicated volume dependence on the Kähler moduli.

Apart from the scenarios discussed here, there exist different alternatives that are not yet well-established in literature, but might eventually lead to breakthrough and cannot be simply neglected. An interesting proposal based on mirror symmetry reasoning is that the superpotential $W$ actually depends also on the Kähler moduli by a new set of fluxes that we have not discovered yet [61]. This way all moduli are fixed already at tree level and the quantum corrections are less important compared to their role in KKLT and LVS. Although reasoning based on conjectures as in this example cannot be appreciated now, some new discoveries in future might give them stronger base. This has happened before in string theory (e.g. the whole topic of flux compactifications).

One more detail we neglected fully here was the existence of open string moduli, coming from the massless spectrum of the open strings that are introduced together with the D-branes. It is generally assumed that these are stabilized heavy, as the dilaton and the complex structure moduli. However, we do not know of explicit stabilization of these moduli in literature at present.

All these issues can only lead us to conclude that the topic of moduli stabilization will be at the forefront of string theory research at least for a couple of years more. With this work we hope to have added something more to the discussion as we tried to argue that stabilization of axionic moduli coming from the $h^{1,1}_{-1}$ part of the orientifold projection deserves more attention and is a nontrivial step in finding the connection between string theory and experiment.
Appendix A

Some explicit Calabi-Yau examples

Let us try to construct some simple examples of Calabi-Yau manifolds following the prescription of [9]. In particular we will restrict our attention to complete intersection manifolds in products of projective spaces. These are given as intersections of surfaces embedded in products of projective spaces of different dimensions. These manifolds are compactly written by configuration matrices, $[\vec{n}] | q$, where $\vec{n}$ is an $m$-dimensional positive integer valued column vector and $q$ an $m \times K$ dimensional non-negative integer valued matrix. $\vec{n}$ specifies the dimensions of the different projective spaces, i.e. the manifold where the Calabi-Yau is embedded is $\mathbb{P}^{n_1} \times \ldots \times \mathbb{P}^{n_m}$. Each column of the matrix $q$ gives the degree of homogeneity of the equation defining a hyper-surface in the above manifold. In the end, the CY manifold is the result of the intersection of all these restrictions, i.e. the set of points for which all these equations hold. In order to make this clear we can give a simple example.

$$\begin{bmatrix}
3 & || & 3 & 0 & 1 \\
3 & || & 0 & 3 & 1
\end{bmatrix} \quad (A.1)$$

represents the system of equations

$$f_{abc}x^ax^bx^c = 0, \ g_{\alpha\beta\gamma}y^\alpha y^\beta y^\gamma, \ h_{aa}x^a y^a = 0 \quad (A.2)$$

in the product space $\mathbb{P}^3 \times \mathbb{P}$. The $x^a$ coordinates belong to the first $\mathbb{P}^3$, while the $y^\alpha$’s belong to the second one. The coefficients $f_{abc}, g_{\alpha\beta\gamma}, h_{aa}$ are not specified and are allowed to take any value. This way we only specify a Calabi-Yau manifold class and not the specific example of it. That is, by specifying a configuration matrix of the form in Eq. (A.1) we specify a full Calabi-Yau moduli space where the moduli are free to take any value, exactly what we need. Clearly the number of dimensions of the resulting manifold equals the dimensions of the initial space minus the number of restricting equations, i.e. $\text{dim} M = \sum_{i=1}^m n_m - K$. Therefore in the example above we have written a Calabi-Yau 3-fold. Note that we implicitly work with complex projective
spaces, so above we calculated the number of complex dimensions. To prove that this is indeed a Calabi-Yau we need to further compute its first Chern class as done in [9] chapter 2 in detail. The conclusion is that we obtain a Calabi-Yau manifold whenever we have that \( \sum_{a=1}^{K} q_a = n_r + 1 \) for each separately. If this condition is not satisfied even for one row the manifold becomes a "del Pezzo" or "Fano" three-fold. Having ensured that the configuration matrix corresponds to a Calabi-Yau manifold, we can calculate its Euler number by the formula expansion:

\[
\chi_E([\vec{n}||\vec{q}]) = \left[ \sum_{r,s,t=1}^{m} \frac{1}{3} \left( \delta^{rst}(n_r + 1) - \sum_{a=1}^{K} q_a^r q_a^s q_a^t \right) J_r J_s J_t \cdot \wedge_a = 1^K \left( \sum_{p=1}^{m} q_a^p J_p \right) \right]_{\text{top}},
\]

where the subscript "top" means one has to take the coefficient in front of \( \prod_{r=1}^{m} J_r \) in the formal expansion of the above formula. This expression gets becomes quite involved for hand calculations rather quickly, but nevertheless gives a well-defined procedure for finding the Euler number.

Additionally to the Euler number we are interested in the elements of the homology and the intersection numbers \( \kappa_{\alpha\beta\gamma} \) for a given CY manifold with a configuration matrix. For certain cases this turns out to be a quite simple task since any nontrivial 4-cycle (i.e. having two complex dimensions) just corresponds to adding an additional constraint equation in the matrix \( \vec{q} \) since this ensures we obtain a sub-manifold of the CY-manifold of codimension 1. To obtain a basis of these cycles is generally a non-trivial task, but in many cases when the matrix \( \vec{q} \) is nice enough (c.f. [9] chapter 8 for more details) one directly obtains that the number \( h^{1,1} \) of non-trivial cycles is equal to the number \( m \) of different projective spaces in the configuration matrix. The basis of cycles is given by adding to the manifold the restriction columns \((1,0,...,0), 0,1,0,...,0,0,0,0,0,1\). We will denote these restrictions by \( X_1, \ldots, X_m \). Therefore \( h^{1,1} = m \) and \( h^{2,1} \) can be found trivially from the Euler number using that \( \chi_E = 2(h^{1,1} - h^{2,1}) \). The triple intersection numbers in this case correspond to the number of intersection point of three of the cycles that form the basis for the homology. Therefore,

\[
\kappa_{rst} = \kappa(X_r, X_s, X_t) = \chi_E([\vec{n}||\vec{q}X_rX_sX_t]).
\]

Since the Euler number for a collection of zero-cells (i.e. points) just counts their total number, this precisely corresponds to the definition of the triple intersection numbers and is thus relatively simple to calculate. Using these rules, here we show explicitly the construction and calculation of the relative topological numbers for a few simple examples of Calabi-Yau three-folds. We use the notation form the main text, i.e. denote the 2-cycle volumes by \( v^i \) and the 4-cycle volumes by \( \tau_i = \frac{1}{2} \kappa_{iab} v^a v^b \).
\[
\begin{bmatrix}
2 & 2 & 1 \\
2 & 2 & 1 \\
1 & 0 & 2
\end{bmatrix}
\]

\[\chi_E = -96; h_{1,1} = 3; h_{2,1} = 51\]

\[Vol(v_i) = 2v_1^2v_2 + v_1^2v_3 + 4v_1v_2v_3 + 2v_1v_2^2 + v_2^2v_3\]

\[\tau_1 = 4v_1v_2 + 2v_1v_3 + 2v_2^2 + 2v_2v_3\]

\[\tau_2 = 2v_1^2 + 4v_1v_2 + 2v_1v_3 + 2v_2v_3\]

\[\tau_3 = v_1^2 + 2v_1v_2 + v_2^2\]

\[Vol(\tau_i) = \frac{1}{128}\tau_3^{5/2}\left[\tau_1^4 - 4\tau_3^3(\tau_2 - \tau_3) + \tau_1^2(6\tau_2^2 - 4\tau_2\tau_3 - 8\tau_3^2) + (\tau_2 - 2\tau_3)^2(\tau_2^2 + 8\tau_2\tau_3 + 20\tau_3^2) - 4\tau_1(\tau_2^3 + \tau_2^2\tau_3 - 4\tau_2\tau_3^2 + 12\tau_3^3)\right]\]

\[
\begin{bmatrix}
3 & 3 & 1 \\
2 & 1 & 2
\end{bmatrix}
\]

\[\chi_E = -114; h_{1,1} = 2; h_{2,1} = 59\]

\[Vol(v_i) = (2v_1^3 + 21v_1^2v_2 + 9v_1v_2^2)/6\]

\[\tau_1 = v_1^2 + 3/2v_2^2 + 7v_1v_2\]

\[\tau_2 = 7/2v_1^2 + 3v_1v_2\]

\[Vol(\tau_i) = \frac{1}{9}\sqrt{\frac{2}{123}}\left(6\tau_1 - 7\tau_2 + \sqrt{9\tau_1^2 - 21\tau_1\tau_2 + 43\tau_2^2}\right) \times \right]

\[
\times \sqrt{-6\tau_1 + 7\tau_2 + 2\sqrt{9\tau_1^2 - 21\tau_1\tau_2 + 43\tau_2^2}}
\]

\[
\begin{bmatrix}
3 & 2 & 1 & 1 \\
3 & 1 & 2 & 1
\end{bmatrix}
\]

\[\chi_E = -106; h_{1,1} = 2; h_{2,1} = 55\]

\[Vol(v_i) = (2v_1^3 + 21v_1^2v_2 + 21v_1v_2^2 + 2v_2^3)/6\]

\[\tau_1 = v_1^2 + 7/2v_2^2 + 7v_1v_2\]

\[\tau_2 = 7/2v_1^2 + 7v_1v_2 + v_2^2\]

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\[ V \text{ol}(\tau_i) = \frac{1}{8625(7\tau_1 - 2\tau_2)} \left( \sqrt{\frac{2}{23}} \sqrt{35\tau_1 - 80\tau_2 + 14\sqrt{5} \sqrt{7\tau_1^2 - 9\tau_1\tau_2 + 7\tau_2^2}} \times \right. \\
\left. \times (-8680\tau_1^2 + 4725\tau_1\tau_2 + 180\tau_2^2 + 1772\sqrt{5}\tau_1 \sqrt{7\tau_1^2 - 9\tau_1\tau_2 + 7\tau_2^2} - \\
\quad -342\sqrt{5}\tau_2 \sqrt{7\tau_1^2 - 9\tau_1\tau_2 + 7\tau_2^2}) \right) \]

\[
\begin{bmatrix}
4 & 2 & 3 \\
1 & 1 & 1
\end{bmatrix}
\]

\[ \chi_E = -226; h_{1,1} = 2; h_{2,1} = 115 \]

\[ V \text{ol}(v_i) = \frac{5}{6}v_1^3 + 3v_1^2v_2 \]

\[ \tau_1 = \frac{5}{2}v_1^2 + 6v_1v_2 \]

\[ \tau_2 = 3v_1^2 \]

\[ V \text{ol}(\tau_i) = \frac{(18\tau_1 - 5\tau_2)\sqrt{\tau_2}}{36\sqrt{3}} \]

\[
\begin{bmatrix}
2 & 3 \\
1 & 2 \\
1 & 2
\end{bmatrix}
\]

\[ \chi_E = -144; h_{1,1} = 3; h_{2,1} = 75 \]

\[ V \text{ol}(v_i) = \frac{1}{2}v_1^2v_2 + v_1^2v_3 + v_1v_2v_3 \]

\[ \tau_1 = v_1v_2 + 2v_1v_3 + 3v_2v_3 \]

\[ \tau_2 = \frac{1}{2}v_1^2 + v_1v_3 \]

\[ \tau_3 = \frac{1}{2}v_1^2 + v_1v_2 \]

\[ V \text{ol}(\tau_i) = \frac{1}{12\sqrt{3}} \left( 6\tau_1 + 3\tau_2 + 3\tau_3 - \sqrt{36\tau_2\tau_3 + (2\tau_1 - \tau_2 + \tau_3)^2} \right) \times \]

\[ \times \sqrt{-2\tau_1 + \tau_2 - \tau_3 + \sqrt{36\tau_2\tau_3 + (2\tau_1 - \tau_2 + \tau_3)^2}} \]

Note that for higher values of \( h^{1,1} \) the inverting of the basis from 2-cycle volumes to 4-cycle volumes becomes harder or impossible to solve analytically and numerical approximations need to be used. However, consistency requires this change of basis to be always possible in principle.
Appendix B

Inverting the Kähler metric

B.1 In the KKLT/LVS procedure

Here we will present in details how to deal with inverting the matrix of partial derivatives $K_{I\bar{J}} = \partial_I \partial_{\bar{J}} K$. We will see that the calculation is simple at tree level, but becomes non-trivial when considering corrections to $K$. Adding only $\alpha'$-corrections we are still able to find full analytic expression for $K^{I\bar{J}}$. We will see in Appendix B.2 that this is no longer the case after considering world-sheet instanton corrections. We consider the tree-level and $\alpha'$-corrections cases separately in different subsections. The notation used throughout the section is as defined for the tree level case. In the following we will completely neglect the complex structure moduli dependence, as they are not coupled to the Kähler moduli and so $K^{I\bar{J}}$ can be found separately in the complex structure dimensions.

B.1.1 Tree level

At tree level,

$$K = -\ln(-i(\tau - \bar{\tau})) - 2\ln(V_{CY})$$

(B.1)

with

$$V_{CY} = \frac{1}{6} \kappa_{\alpha\beta\gamma} t^\alpha t^\beta t^\gamma = \frac{1}{3} \tau_\alpha t^\alpha,$$

(B.2)

where summation over repeated indices is implied. This is the standard expression for the volume of the CY manifold in terms of the 2-cycle volumes $t^\alpha$ (also denoted $v^\alpha$ in the main text) and the corresponding 4-cycle volumes $\tau_\alpha = 1/2\kappa_{\alpha\beta\gamma} t^\beta t^\gamma$. We now introduce the Kähler moduli variables which we will use from now on. They are implicitly defined by:

$$\tau_\alpha \equiv -3i(\rho_\alpha - \bar{\rho}_\alpha),$$

(B.3)
such that \( V_{CY} = -i(\rho_\alpha - \bar{\rho}_\alpha)t^\alpha \). Remember that \( t^\alpha \) are also functions of our new variables via \( \kappa_{\alpha\beta\gamma}t^\beta t^\gamma = -6i(\rho_\alpha - \bar{\rho}_\alpha) \). Another important definition that will be crucial in the following, is:

\[
\kappa_{\alpha\beta} \equiv \kappa_{\alpha\beta\gamma}t^\gamma. \tag{B.4}
\]

Let us then denote the inverse \((\kappa_{\alpha\beta})^{-1} = \kappa^{\alpha\beta}\). We will never need to invert this matrix explicitly, all we use \( \kappa_{\alpha\beta} \) for is to be able to write explicitly the volume of the manifold and its derivatives in terms of the variables (B.3). Now,

\[
\tau_\alpha = \frac{1}{2}\kappa_{\alpha\beta}t^\beta \Rightarrow t^\alpha = 2\kappa_{\alpha\beta} \tau_\beta \Rightarrow \partial_{\tau_\alpha}t^\beta = \kappa_{\alpha\beta\gamma}t^\gamma = \kappa_{\alpha\beta} \Rightarrow \frac{\partial t^\alpha}{\partial \tau_\beta} = \kappa^{\alpha\beta}. \tag{B.5}
\]

Note how we use the fact that \( \kappa_{\alpha\beta} = \kappa_{\beta\alpha} \). Now we can obtain the important identities:

\[
\frac{\partial t^\alpha}{\partial \rho_\beta} = \frac{\partial t^\alpha}{\partial \tau_\gamma} \frac{\partial \tau_\gamma}{\partial \rho_\beta} = -3i\kappa^{\alpha\beta} \quad \frac{\partial t^\alpha}{\partial \bar{\rho}_\beta} = 3i\kappa^{\alpha\beta} \tag{B.6}
\]

and

\[
\frac{\partial V_{CY}}{\partial \rho_\alpha} = \frac{\partial \tau_\beta t^\beta + \tau_\beta \frac{\partial t^\beta}{\partial \rho_\alpha}}{\partial \rho_\alpha} = -it^\alpha - 3it^{\alpha\beta}t_\beta = -\frac{3}{2}it^\alpha \quad \frac{\partial V_{CY}}{\partial \bar{\rho}_\alpha} = \frac{3}{2}it^\alpha. \tag{B.7}
\]

Equipped with all this machinery, we are now ready to tackle out initial problem. From (B.1) we get

\[
K_\tau = -\frac{1}{\tau - \bar{\tau}}, \quad K_\rho_\alpha = \frac{3it^\alpha}{V_{CY}}, \tag{B.8}
\]

where as usual lower indices of \( K \) denote partial derivatives. Then,

\[
K_{\tau\bar{\tau}} = -\frac{1}{(\tau - \bar{\tau})^2}, \quad K_{\tau\rho_\alpha} = K_{\rho_\alpha\bar{\tau}} = 0, \quad K_{\rho_\alpha\bar{\rho}_\beta} = \frac{\partial}{\partial \bar{\rho}_\beta} \frac{3it^\alpha}{V_{CY}} = -9\kappa^{\alpha\beta} \frac{V_{CY}}{2(V_{CY})^2} + \frac{9t^{\alpha\beta}}{2(V_{CY})^2}, \tag{B.9}
\]

Due to vanishing of \( K_{\tau\rho_\alpha} \) and \( K_{\rho_\alpha\bar{\tau}} \) our problem simplifies considerably since now \( K_{\tau\bar{\tau}} = (K_{\tau\bar{\tau}})^{-1} = -(\tau - \bar{\tau})^2 \) (which is irrelevant for the full potential since \( D_\tau W = 0 \)) and \( K_{\rho_\alpha\bar{\rho}_\beta} = (K_{\rho_\alpha\bar{\rho}_\beta})^{-1} \). Therefore we just need to construct our inverse to satisfy

\[
K_{\rho_\alpha\bar{\rho}_\beta} K^{\rho_\alpha\bar{\rho}_\gamma} = \delta^\alpha_\gamma. \tag{B.10}
\]

It is easy to make an ansatz for \( K^{\rho_\alpha\bar{\rho}_\beta} \) since it has to have two free lower indices that are contracted with the upper indices of \( \kappa^{\alpha\beta} \) and \( t^\alpha t^\beta \). We thus have two possible choices
and so generally our inverse matrix looks like $K^{\rho_\alpha \bar{\rho}_\gamma} = a\kappa_{\beta\gamma} + b\tau_{\beta}\tau_{\gamma}$ with coefficients $a$ and $b$ with no free indices to be determined by (B.10). Explicitly,

$$
\left( -\frac{9\kappa_{\alpha\beta}}{V_{CY}} + \frac{9t^\alpha t^\beta}{2(V_{CY})^2} \right) (a\kappa_{\beta\gamma} + b\tau_{\beta}\tau_{\gamma}) = -\frac{9a}{V_{CY}}\delta^\alpha_{\gamma} + \left( \frac{9a}{(V_{CY})^2} - \frac{9b}{2V_{CY}} + \frac{27b}{2V_{CY}} \right) t^\alpha t_{\gamma}.
$$

(B.11)

Clearly only the first term is proportional to $\delta^\alpha_{\gamma}$ while the second term has to vanish in order to satisfy (B.10). Therefore, $a = -V_{CY}/9$ and

$$
-1 + 9b = 0 \Rightarrow b = \frac{1}{9},
$$

such that we obtain the full analytic expression for the inverse matrix:

$$
K^{\rho_\alpha \bar{\rho}_\beta} = -\frac{V_{CY}}{9}\kappa_{\alpha\beta} + \frac{1}{9}\tau_{\alpha}\tau_{\beta}.
$$

(B.13)

Now we can show the famous no-scale structure of the full potential, since at tree level (after fixing the complex structure and dilaton moduli) it is given by

$$
V = e^K \left( K^{\rho_\alpha \bar{\rho}_\beta} K_{\rho_\alpha} K_{\bar{\rho}_\beta} - 3 \right) |W_0|^2.
$$

(B.14)

So

$$
K^{\rho_\alpha \bar{\rho}_\beta} K_{\rho_\alpha} K_{\bar{\rho}_\beta} = \left( -\frac{V_{CY}}{9}\kappa_{\alpha\beta} + \frac{1}{9}\tau_{\alpha}\tau_{\beta} \right) \frac{3it^\alpha - 3it^\beta}{V_{CY}}
$$

$$
\left( -V_{CY}\kappa_{\alpha\beta} + \tau_{\alpha}\tau_{\beta} \right) \frac{t^\alpha t^\beta}{V_{CY}^2} = -6 + 9 = 3,
$$

(B.15)

using identity (B.2). Therefore the two terms in (B.14) cancel each other and $V = 0$ in the direction of the Kähler moduli as expected.

As seen our method rigourously proves the no-scale structure of the tree-level potential. The derivation is slightly more laborious as compared to standard literature, but our method is worth explaining in detail since it can be very easily generalized to include corrections to the Kähler potential. As the reader will see in the next subsection the problem of inverting the matrix in the case of the Large Volume Scenario involves only slightly bigger calculational effort.

### B.1.2 $\alpha'$-corrections

The Kähler potential with $\alpha'$ corrections was first derived in [24] and reads:

$$
K = -\ln(2S_1) - 2\ln(V_{CY} + \frac{\epsilon}{2}S_1^{3/2}),
$$

(B.16)
where we follow the notation of [28] to write $S_1 = \text{Re}S$ as shorthand for $(\frac{\tau - \bar{\tau}}{2i})$. In the following for calculational simplicity we will perform a shift $\tau/2 \rightarrow \tau$ which does not influence the final result as it is written in terms of the same $S_1$, now defined as $(-i(\tau - \bar{\tau}))$.

The form of $K^{IJ}$ in this case is well-known from literature [24, 26] and one can find it along the lines of the previous subsection taking (B.16) instead of (B.1). Here we will show the derivation of the same result in a somewhat roundabout way. Since we always have in mind that all formulae are valid for large volumes of the CY manifold, we will perform an expansion of $K$ in powers of $V_{\text{CY}}$ and take only the leading term of the $\alpha'$ correction as all other terms are suppressed. Therefore,

$$K = -\ln(2S_1) - 2\ln(V_{\text{CY}}) - \frac{\xi S_1^{3/2}}{V_{\text{CY}}}, \quad \text{(B.17)}$$

Clearly we cannot expect to obtain the exact same result for $K^{IJ}$ from (B.17) as compared to the one from (B.16) but we know they must coincide in the first two leading terms in powers of $V_{\text{CY}}$. We will soon see that this is indeed correct. Let us first calculate $K^{I\bar{J}}$:

$$K_{\tau \bar{\tau}} = -\frac{1}{\tau - \bar{\tau}} + \frac{3i\xi(-i(\tau - \bar{\tau}))^{1/2}}{2V_{\text{CY}}},$$

$$K_{\rho \alpha} = \frac{3i\xi(-i(\tau - \bar{\tau}))^{3/2}t^\alpha}{2V_{\text{CY}}^2}, \quad \text{(B.18)}$$

leading to

$$K_{\tau \bar{\tau}} = -\frac{1}{(\tau - \bar{\tau})^2} + \frac{3\xi}{4(-i(\tau - \bar{\tau}))^{1/2}V_{\text{CY}}} = \frac{4V_{\text{CY}} - 3S_1^{3/2}}{4V_{\text{CY}}^2},$$

$$K_{\rho \alpha} = K_{\rho \alpha \tau} = \frac{9\xi(-i(\tau - \bar{\tau}))^{1/2}t^\alpha}{4V_{\text{CY}}^{3/2}} = \frac{9\xi S_1^{1/2}t^\alpha}{4V_{\text{CY}}^2}. \quad \text{(B.19)}$$

$$K_{\rho \alpha \bar{\beta}} = -\frac{9\kappa^{\alpha \beta}}{V_{\text{CY}}} + \frac{9t^\alpha t^\beta}{2(V_{\text{CY}})^2} + \frac{9\xi(-i(\tau - \bar{\tau}))^{3/2}K^{\alpha \beta}}{2V_{\text{CY}}^2} - \frac{9\xi(-i(\tau - \bar{\tau}))^{3/2}t^\alpha t^\beta}{2V_{\text{CY}}^3} \kappa^{\alpha \beta} + \frac{9\xi S_1^{3/2}}{2V_{\text{CY}}^{3/2}} - \frac{9\xi S_1^{3/2}}{2V_{\text{CY}}^{3/2}}t^\alpha t^\beta.$$

Here we encounter off-diagonal elements which means that now (unlike the case at tree-level) the matrix $K^{I\bar{J}}$ is not block diagonal and we have to invert the whole matrix. However, this is not a real problem since we have only one dilaton modulus, i.e. $K_{\tau \bar{\tau}}$ is 1-dimensional. We know;

$$\begin{pmatrix} K_{\tau \bar{\tau}} & K_{\tau \bar{\rho} \beta} \\ K_{\rho \alpha \bar{\tau}} & K_{\rho \alpha \bar{\beta}} \end{pmatrix} \begin{pmatrix} K_{\tau \bar{\tau}} & K_{\tau \bar{\rho} \gamma} \\ K_{\rho \alpha \bar{\tau}} & K_{\rho \alpha \bar{\beta} \gamma} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \delta^\gamma_\alpha \end{pmatrix}, \quad \text{(B.20)}$$

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which can be rewritten as two independent systems of equations for the first and the second row of the inverse matrix. Since we are interested only in $K^{\rho\alpha\bar{\rho}}\bar{\alpha}$ we will focus on the second row. The two coupled equations are

$$K^{\rho\alpha\bar{\rho}}K_{\tau\tau} + K^{\rho\alpha\bar{\rho}}K_{\rho\alpha\bar{\rho}} = 0$$

$$K^{\rho\alpha\bar{\rho}}K_{\tau\rho\alpha} + K^{\rho\alpha\bar{\rho}}K_{\rho\alpha\bar{\rho}} = \delta^\gamma_\alpha. \quad (B.21)$$

From the first one we find $K^{\rho\alpha\bar{\rho}}K_{\tau\rho\alpha} = -K^{\rho\alpha\bar{\rho}}K_{\rho\alpha\bar{\rho}}(K_{\tau\tau})^{-1}$ and substituting in the second one we finally obtain

$$K^{\rho\alpha\bar{\rho}}(K_{\rho\alpha\bar{\rho}} - K_{\rho\alpha\bar{\rho}}(K_{\tau\tau})^{-1}K_{\tau\rho}) = \delta^\gamma_\alpha. \quad (B.22)$$

So we are again facing an equation of type similar to (B.10) and we already know what to do. The ansatz for $K^{\rho\alpha\bar{\rho}}$ is the same as before and therefore:

$$\left(\frac{-18V_{CY} + 9\xi S_1^{3/2}}{2V_{CY}^2}\kappa^{\beta\beta} + \frac{9V_{CY} + 9\xi S_1^{3/2}}{2V_{CY}^3}\tau^\beta\tau^\gamma - \frac{9^2\xi^2 S_1^3}{4V_{CY}^2(4V_{CY} - 3S_1^{3/2})}\right) \times \left(a\kappa_{\alpha\beta} + b\tau_\alpha\tau_\beta\right) = \delta^\gamma_\alpha. \quad (B.23)$$

Therefore we again have two terms, one of them proportional to $\kappa_{\alpha\beta}\kappa^{\beta\gamma} = \delta^\gamma_\alpha$ such that we obtain

$$a = \frac{2V_{CY}^2}{-18V_{CY} + 9\xi S_1^{3/2}}. \quad (B.24)$$

Requiring the second term to vanish we get

$$(2a + 3bV_{CY}) \left(\frac{9V_{CY} + 9\xi S_1^{3/2}}{2V_{CY}^3} - \frac{9^2\xi^2 S_1^3}{4V_{CY}^2(4V_{CY} - 3S_1^{3/2})}\right) + \frac{-18V_{CY} + 9\xi S_1^{3/2}b}{4V_{CY}^2} = 0 \quad (B.25)$$

and after somewhat cumbersome calculation (which is easily solved by computer software)

$$b = \frac{4V_{CY}(8V_{CY}^2 - (6 + 8\xi)S_1^{3/2}V_{CY} + (6\xi - 9\xi^2)S_1^3)}{9(2V_{CY} - S_1^{3/2}\xi)(16V_{CY}^2 - (12 + 20\xi)S_1^{3/2}V_{CY} + (15\xi - 27\xi^2)S_1^3)}. \quad (B.26)$$

We can check if the expressions for $a$ and $b$ are consistent by comparing their leading term at large volume with the answers at tree-level. As expected, for $V_{CY} \to \infty$, $a = -V_{CY}/9$ and $b = 1/9$. This is the case since the $\alpha'$ corrections are only subleading in volume and contribute to $a$ and $b$ in lower powers of $V_{CY}$. Therefore one might think that our calculation was useless since we eventually care about the leading terms. However, as shown above, the leading order in the full potential $V$ is cancelled by the
term $3|W_0|^2$ and therefore the former subleading terms become leading. Explicitly, using (B.24), (B.26) and (B.18):
\[
K^\rho_{\bar{\rho}} \partial_\rho \partial_{\bar{\rho}} K - 3 = \frac{3\xi S_1^{3/2}(-4V_{CY}^2 + (3 - 31\xi)S_1^{3/2}V_{CY} + 3\xi S_1^3)}{V_{CY}(-16V_{CY}^2 + 4(3 + 5\xi)S_1^{3/2}V_{CY} + 3(-5\xi + 9\xi^2)S_1^3)}.
\] (B.27)

Now we are really only interested in the leading term of (B.27), which is $\frac{3\xi S_1^{3/2}}{V_{CY}}$. As discussed after (B.17) we expect the first two leading terms to coincide with the exact expression. The first leading term cancels and we can see that the next leading term is indeed the same as expected (cf. (3.31) of [24]). Therefore we explicitly showed what was obvious from the very beginning, namely that the expansion of $K$ as in (B.17) can be used safely to determine the leading order terms of the moduli potential. In the case at hand the expansion actually makes calculations more involved, but after adding more corrections to the Kähler potential this approach will turn out to be more useful in comparing the relative importance of different correction terms.

### B.2 The full Kähler metric

Here we will present in details how to deal with inverting the matrix of partial derivatives $K_{IJ} = \partial_I \partial_J K$ after including the non-geometric Kähler moduli. We will show that, although rather non-trivial, there is exact analytic solution for the inverse metric both at tree level and with $\alpha'$-corrections included. We will therefore consider these cases separately in different subsections. We will again neglect the complex structure moduli dependence, as they are not coupled to the Kähler moduli and dilaton in $K$.

#### B.2.1 Tree level

More explicitly, the Kähler potential (3.6) is
\[
K = -\ln(-i(\tau - \bar{\tau})) - 2\ln\left(\frac{(\frac{3}{2}(T_\alpha - \bar{T}_\alpha) - \frac{i}{2(\tau - \bar{\tau})}K_{aab}(G - \bar{G})^a(G - \bar{G})^b)v^a(T_\alpha, G^a, \tau)}{6}\right),
\] (B.28)

where $v^a$ is an implicit function of the Kähler oordinates. It is given by the relation $\kappa_\alpha = \kappa_{\alpha\beta\gamma}v^\beta v^\gamma = \kappa_{\alpha\beta}v^\beta$, where we made the definition $\kappa_{\alpha\beta} \equiv \kappa_{\alpha\beta\gamma}v^\gamma$ as in section B.1. Therefore, $v^\alpha = \kappa^{\alpha\beta}K_{\beta\gamma}$. We make an analogous definition for the intersection numbers with Latin indices: $\kappa_{ab} \equiv \kappa_{aabc}v^a$.

Now we can calculate the actual Kähler metric, using the following matrix definitions that one can use for shorthand and easier calculation:
\[
G^{\alpha\beta} \equiv -\frac{2}{3}K\kappa^{\alpha\beta} + 2v^\alpha v^\beta, \quad G_{ab} \equiv -\frac{3}{2}\kappa_{ab}\kappa,
\] (B.29)
and their corresponding inverses

\[ G_{\alpha\beta} = \frac{3}{2} \left( \frac{\kappa_{\alpha\beta}}{\kappa} - \frac{3}{2} \kappa_{\alpha} \kappa_{\beta} \right), \quad G^{ab} = -\frac{2}{3} \kappa K^{ab}. \tag{B.30} \]

The first partial derivatives of \( K \) can then be computed to be:

\[
K_{\tau} = -K_{\bar{\tau}} = -\frac{1}{\tau - \overline{\tau}} - \frac{3i}{2(\tau - \overline{\tau})^2} \kappa^{ab} (G - \overline{G})^a (G - \overline{G})^b = i e^\phi \frac{2}{\kappa} + i G_{ab} b^a b^b, \]

\[
K_{G} = -K_{\bar{G}} = \frac{3i}{(\tau - \overline{\tau}) \kappa} \kappa^{ab} (G - \overline{G})^b = 2i G_{ab} b^b, \tag{B.31} \]

\[
K_{T} = -K_{\bar{T}} = \frac{2}{\kappa} e^\phi, \]

where we used from (3.4) that \((\tau - \overline{\tau}) = 2i e^{-\phi} \) and \((G - \overline{G})^a = -(\tau - \overline{\tau}) b^a = -2i e^{-\phi} b^a\).

Then,

\[
K_{\tau\bar{\tau}} = \frac{e^{2\phi}}{4} + e^\phi G_{ab} b^a b^b + \frac{9}{16\kappa^2} G^{\alpha \beta} \kappa_{\alpha ab} b^a b^b \kappa_{\beta cd} b^c b^d, \]

\[
K_{\bar{G}\bar{G}} = K_{\tau G} = e^\phi G_{ab} b^b + \frac{9}{8\kappa^2} G^{\alpha \beta} \kappa_{\alpha ab} b^b \kappa_{\beta cd} b^c b^d, \]

\[
K_{\tau T} = -K_{\bar{T} \tau} = -\frac{3i}{4\kappa^2} G^{\alpha \beta} \kappa_{\beta ab} b^a b^b, \]

\[
K_{G \bar{G}} = = e^\phi G_{ab} + \frac{9}{4\kappa^2} G^{\alpha \beta} \kappa_{\alpha ab} t^c \kappa_{\beta cd} \bar{b}^d, \tag{B.32} \]

\[
K_{T \bar{T} \alpha} = = -K_{G a \bar{T}} = -\frac{3i}{2\kappa^2} G^{\alpha \beta} \kappa_{\beta ab} b^b, \]

\[
K_{T \bar{T} \beta} = -G^{\alpha \beta}. \]

The inverse metric can be found after a lengthy calculation, which goes as follows. One can make an ansatz for each of the elements of the inverse metric from the number of free indices, e.g. the component \( K_{T \bar{T} \tau} \) has only one free lower index \( \alpha \) as opposing to the upper index of the original metric component. Therefore, a possible ansatz could be \( K_{T \alpha \tau} = a \kappa_{\alpha} + b \kappa_{ab} b^a b^b \), where \( a \) and \( b \) can be any expression with fully contracted indices (or with no indices at all). Plugging the ansatz for every element of the inverse matrix leads to 9 coupled equations (see App. B.1 for more details) which lead to unique determination of all undetermined values. For the specific example, we find
$a = 0, \ b = 3ie^{-\phi}$. Explicitly, the whole inverse metric is:

\[
K^{\tau\bar{\tau}} = 4e^{-2\phi}, \\
K^{G_a\bar{G}_a} = -4e^{-2\phi}b^a, \\
K^{T_a\bar{T}_a} = -K^{\tau\bar{T}_a} = 3ie^{-2\phi}\kappa_{aabc}b^c b^b, \\
K^{G_a\bar{G}_a} = e^{-\phi}G^{ab} + 4e^{-2\phi}b^a b^b, \\
K^{T_a\bar{G}_a} = -3ie^{-\phi}2G^{ab}\kappa_{aabc}b^c b^a + 9e^{-2\phi}4\kappa_{aabb}b^ab^b b^b.
\]

(B.33)

Having found the inverse metric and the first partial derivatives (B.31), to obtain Eq. (3.9) is down to some trivial algebra. However, it might be quite interesting in which way one gets the number 4. If we break up the sum into two parts - a sum which runs over all $T_a$ and $G^a$ but not over the dilaton, plus the remainder of the whole sum (i.e. where at least one of the indices goes over $\tau$). So,

\[
K^{ij}_{\tau\bar{\tau}} = 3 + 9e^{-2\phi}\left(\kappa_{aabb}v^ab^b\right)^2, \quad i, j = T_1...T_h, G_{1...G_h}, \quad G_{(1,1)}^{(1,1)}, (B.34)
\]

while for the remainder one gets $1 - 9e^{-2\phi}\left(\kappa_{aabb}v^ab^b\right)^2$ as expected since the two sums add up to 4.

B.2.2 $\alpha'$-corrections

In order to simplify notation, we first use the following definitions:

\[
\hat{\xi} \equiv \frac{\xi}{2(2i)^{3/2}}, \\
Y \equiv V_{CY} + \frac{\xi}{2} \left(\frac{\tau - \bar{\tau}}{2i}\right)^{3/2} = \frac{\kappa}{6} + \hat{\xi}(\tau - \bar{\tau})^{3/2}. (B.35)
\]

In the following we will drop the hat of $\hat{\xi}$ and will use this new definition until the end of the section where we switch to the proper definition. With these identifications, the Kähler potential takes a misleadingly simple form:

\[
K = -\ln(-i(\tau - \bar{\tau})) - \ln(Y). \quad (B.36)
\]

However, $Y$ is now dependent on all variables. Its partial derivatives are:

\[
\frac{\partial Y}{\partial T^a_\alpha} = \frac{v^a}{6}, \\
\frac{\partial Y}{\partial G^a} = -\frac{i}{4(\tau - \bar{\tau})}\kappa_{aabc}(G - \bar{G})^b, \\
\frac{\partial Y}{\partial \tau} = \frac{i}{8(\tau - \bar{\tau})^2}\kappa_{aabc}(G - \bar{G})^a(G - \bar{G})^b + \frac{3}{2}\hat{\xi}(\tau - \bar{\tau})^{1/2}; (B.37)
\]
where we define $\kappa_{ab}, \kappa_{\alpha\beta}$ as in the previous subsection. However, we slightly change the definition of $G^{\alpha\beta}, G_{ab}$:

$$G^{\alpha\beta} = -\frac{Y}{9} \kappa^{\alpha\beta} + \frac{1}{18} v^\alpha v^\beta, \quad G_{ab} = -\frac{\kappa_{ab}}{4Y}. \quad (B.38)$$

Their corresponding inverses are

$$G_{\alpha\beta} = -\frac{9\kappa_{\alpha\beta}}{Y} + \frac{\kappa_{\alpha\kappa}\kappa_{\beta\lambda}}{2Y \left(-\frac{Y}{9} + \frac{\kappa}{18}\right)}, \quad G^{ab} = -\frac{2}{3} \kappa^{ab}. \quad (B.39)$$

With these, and using $(\tau - \bar{\tau}) = 2ie^{-\phi}$, $(\tau - \bar{\tau}) b^a = -2ie^{-\phi}b^a$,

$$K_\tau = -K_{\bar{\tau}} = -\frac{1}{\tau - \bar{\tau}} - \frac{i}{4(\tau - \bar{\tau})^2} \kappa_{ab}(G - \bar{G})^a(G - \bar{G})^b - \frac{3\xi(\tau - \bar{\tau})^{1/2}}{Y} = \frac{i e^\phi}{2} + i G_{ab} b^a b^b - \frac{3\xi(2i)^{1/2} e^{-\phi/2}}{Y},$$

$$K_{G^a} = -K_{\bar{G}^a} = \frac{i}{2(\tau - \bar{\tau})} \kappa_{ab}(G - \bar{G})^b = 2iG_{ab} b^b,$$

$$K_{T^a} = K_{\bar{T}^a} = -\frac{v^a}{3Y}. \quad (B.40)$$

The metric then takes the form:

$$K_\tau = \left(\frac{e^{2\phi}}{4} + \frac{3\xi e^{\phi/2}}{2(2i)^{1/2} Y} - \frac{9i e^{-\phi}}{2Y^2} \right) + G_{ab} b^a b^b + \frac{9}{16Y^2} \kappa^{\alpha\beta} \kappa_{\alpha\alpha} b^\beta b^\beta, \quad \kappa_{\beta\beta} b^\beta b^\beta, \quad (B.41)$$

$$K_{G^a} = K_{\bar{G}^a} = \left(\frac{e^{\phi}}{2} + \frac{3i \xi e^{\phi/2}}{Y} \right) G_{ab} b^a b^b + \frac{9}{8Y^2} \kappa^{\alpha\beta} \kappa_{\alpha\alpha} b^\beta b^\beta, \quad \kappa_{\beta\beta} b^\beta b^\beta,$$

$$K_{T^a} = K_{\bar{T}^a} = -\frac{3i}{4Y^2} G^{\alpha\beta} \kappa_{\alpha\beta} b^\beta b^\beta - \frac{\xi(2i)^{1/2} e^{-\phi/2}}{2Y^2} v^a,$$

$$K_{G^a} = K_{\bar{G}^a} = e^\phi G_{ab} + \frac{9}{4Y^2} G^{\alpha\beta} \kappa_{\alpha\alpha} b^\beta b^\beta \kappa_{\beta\beta} b^\beta b^\beta,$$

The inverse metric is found along the procedure from the previous section, described after Eq.(B.32). For easier reading, we will write down the ansatz for the inverse metric.
and give the resulting prefactors separately.

\[ K^{\tau\bar{\tau}} = a, \]

\[ K^{G^{\tau\bar{G}}} = b\beta^2, \]

\[ K^{T_{\alpha}\bar{T}_{\alpha}} = -K^{T_{\alpha}T_{\alpha}} = c\kappa_{\alpha\beta}b^\alpha b^\beta + d\kappa_{\alpha}, \]

\[ K^{G^{\tau}G^{\beta}} = IG^{ab} + ml^\alpha b^\beta, \]

\[ K^{T_{\alpha}\bar{G}^{\alpha}} = -K^{G^{\tau}T_{\alpha}} = -eG^{ab}\kappa_{\alpha\beta}b^\beta - f\kappa_{\alpha\beta}b^\beta b^\alpha - f'\kappa_{\alpha}b^\alpha, \]

\[ K^{T_{\alpha}T_{\beta}} = gG_{\alpha\beta} + hG^{ab}\kappa_{\alpha\alpha}b^\alpha b^\beta \]

\[ + j_1\kappa_{\alpha\beta}b^\alpha b^\beta \kappa_{\beta\alpha}b^\alpha b^\beta + j_2(\kappa_{\alpha\beta}b^\alpha b^\beta + \kappa_{\alpha\beta}b^\alpha b^\beta \kappa_{\beta\alpha}) + j_4\kappa_{\alpha\beta}. \]

The corresponding prefactors are

\[ a = \frac{2}{2(-\frac{Y}{9} + \frac{\kappa}{18})} \left( \frac{e^{2\phi}}{4} + \frac{3\xi e^{\phi/2}}{2(2i)^{1/2}Y} - \frac{9i\xi^2 e^{-\phi}}{Y^2} \right) + \frac{in\xi e^{-\phi}}{Y^2}, \]

\[ b = -a \]

\[ c = \frac{3i}{2(-\frac{Y}{9} + \frac{\kappa}{18})} \left( \frac{e^{2\phi}}{4} + \frac{3\xi e^{\phi/2}}{2(2i)^{1/2}Y} - \frac{9i\xi^2 e^{-\phi}}{Y^2} \right) + \frac{in\xi e^{-\phi}}{Y^2}, \]

\[ d = -\frac{(2i)^{1/2}e^{-\phi/2}}{2(-\frac{Y}{9} + \frac{\kappa}{18})} \left( \frac{e^{2\phi}}{4} + \frac{3\xi e^{\phi/2}}{2(2i)^{1/2}Y} - \frac{9i\xi^2 e^{-\phi}}{Y^2} \right) + \frac{in\xi e^{-\phi}}{Y^2}, \]

\[ e = \frac{3i}{2}e^{-\phi}, \]

\[ f = c \]

\[ f' = d \]

\[ g = Y^2, \]

\[ h = \frac{9}{4}e^{-\phi}, \]

\[ j_1 = \frac{9}{8}\left( -\frac{Y}{9} + \frac{\kappa}{18} \right) \left( \frac{e^{2\phi}}{4} + \frac{3\xi e^{\phi/2}}{2(2i)^{1/2}Y} - \frac{9i\xi^2 e^{-\phi}}{Y^2} \right) + \frac{in\xi e^{-\phi}}{Y^2}, \]

\[ j_2 = \frac{3i(2i)^{1/2}e^{-\phi}}{4\left( -\frac{Y}{9} + \frac{\kappa}{18} \right)} \left( \frac{e^{2\phi}}{4} + \frac{3\xi e^{\phi/2}}{2(2i)^{1/2}Y} - \frac{9i\xi^2 e^{-\phi}}{Y^2} \right) + \frac{in\xi e^{-\phi}}{Y^2}, \]

\[ j_4 = \frac{-2i\xi^2 e^{-\phi}}{\left( -\frac{Y}{9} + \frac{\kappa}{18} \right)} \left( -\frac{Y}{9} + \frac{\kappa}{18} \right) \left( \frac{e^{2\phi}}{4} + \frac{3\xi e^{\phi/2}}{2(2i)^{1/2}Y} - \frac{9i\xi^2 e^{-\phi}}{Y^2} \right) + \frac{in\xi e^{-\phi}}{Y^2}, \]

\[ t = e^{-\phi}, \]

\[ m = a. \]
Clearly the inverse metric in this form is not very suitable for calculational purposes. As we are interested in the large volume behavior, we can expand the coefficients $a, ..., m$ and take the leading terms in the limit where $V_{CY} \to \infty$. In order to calculate $K^{ij} K_i K_j$ exactly upto $O(V_{CY}^{-5/3})$ we also need some of the subleading terms of inverse metric. With this choice of relevant accuracy, we obtain:

\begin{align}
K^{\tau \bar{\tau}} & \approx 4 e^{-2\phi} - \frac{24 \xi e^{-7\phi/2}}{(2i)^{1/2} V_{CY}}, \\
K^{G^a \bar{\tau}} &= K^{\tau G^a} \approx -4b^a, \\
K^{T^a \bar{\tau}} &= -K^{\tau T^a} \approx 3i e^{-2\phi} \kappa_{a a b} b^a b^b - \frac{9\xi (2i)^{1/2} e^{-5\phi/2}}{V_{CY}} \kappa_a, \\
K^{G^a \bar{G}^b} &\approx e^{-\phi} G^{a b} + 4e^{-2\phi} b^a b^b, \\
K^{T^a \bar{G}^b} &= -K^{G^a T^b} \approx -\frac{3i e^{-\phi}}{2} G^{a b} \kappa_{a b c} b^c - \\
&\quad - 3i e^{-2\phi} \kappa_{a b c} b^c b^c b^a + \frac{9\xi (2i)^{1/2} e^{-5\phi/2}}{V_{CY}} \kappa_a b^a, \\
K^{T^a T^b} &\approx Y^2 G_{a b} + \frac{9e^{-\phi}}{4} G^{a b} \kappa_{a a b} b^a b^b + \frac{9e^{-2\phi}}{4} \kappa_{a a b} b^a b^b \kappa_{b c d} b^c b^d + \\
&\quad + \frac{27i\xi (2i)^{1/2} e^{-5\phi/2}}{4 V_{CY}} (\kappa_a \kappa_{b a b} b^a b^b + \kappa_{a a b} b^a b^b \kappa_{b}) - \frac{81 i \xi^2 e^{-3\phi}}{V_{CY}^2} \kappa_a \kappa_b.
\end{align}

Now we can again calculate $K^{ij} K_i K_j$ and we find it again equal to 4 as in the tree level case Eq.(3.9). This time the 4 comes as follows:

\begin{align}
K^{ij} K_i K_j &= 3 + \frac{e^K e^{-2\phi}|W|^2}{4 V_{CY}^2} (\kappa_{a a b} b^a b^a b^b)^2 - \frac{6 \xi e^{-3\phi/2}}{(2i)^{1/2} V_{CY}} + O(V_{CY}^{-5/3}), \\
i, j &= T_1 ... T_{h_t^{(1,1)}}, G^1 ... G^{h_t^{(1,1)}},
\end{align}

and the remainder is what is left such that it sums to 4. Note that we still have $\hat{\xi}$ dependence, and if we switch to $\xi$ as in literature we recover the standard term that appears in literature (c.f. (17) of [21]):

\begin{align}
- \frac{6 \xi e^{-3\phi/2}}{(2i)^{1/2} V_{CY}} = \frac{3 \xi e^{-3\phi/2}}{4 V_{CY}}.
\end{align}
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