Recent progress in holography

Marika Taylor

University of Amsterdam

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Outline

- **A brief introduction to holography**
- Holography as a tool for strongly correlated systems

What is Holography?

The physics of the 20th century was founded on two pillars:

- Quantum theory
- General relativity

But combining them to obtain a quantum theory of gravity is a longstanding problem.

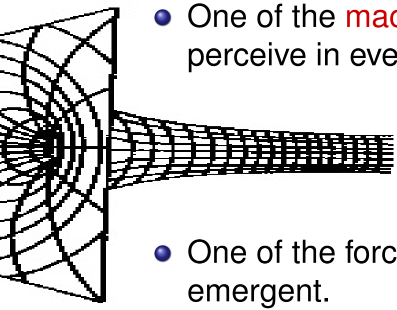
What is Holography?

In recent years we have learnt that this difficulty may reflect the need for a *new conceptual framework*:

Holography Any quantum theory of gravity should have a description in terms of a quantum field theory (QFT), *without gravity*, in one dimension less.

Holography: a paradigm shift

Holography is an idea unlike anything seen before. It implies:

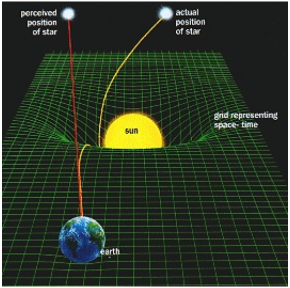


- One of the macroscopic dimensions we perceive in everyday life is **emergent**.
- One of the forces of Nature, gravity, is also emergent.

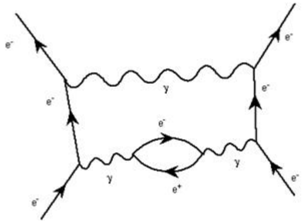
The holographic dictionary

A detailed holographic dictionary relates the gravitational physics to the theory with no gravity:

Gravity in three dimensions

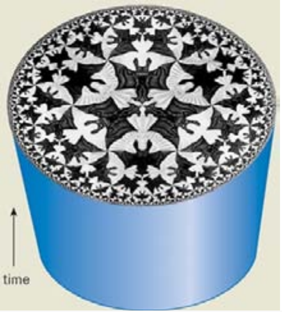


Quantum fields in flatland



Important features of the holographic dictionary

The shape and curvature of the spacetime reflect the properties of the flatland theory. The extra dimension captures the behavior of the flatland theory under scale transformations.



E.g. If the flatland theory is scale invariant (a CFT), the spacetime has negative curvature (an Anti-de Sitter space).

Important features of the holographic dictionary

Every flatland computable can be mapped into a computable quantity in the gravity theory and vice versa.

- Simple calculations on one side can be complicated on the other: F = mg!
- Many interesting QFT computables (correlation functions, transport properties) *are* computable from probe ripples of the spacetime.

Important features of the holographic dictionary

If g is the coupling constant of the flatland theory, the coupling constant of the gravity theory is $1/g$.

- This implies that a strongly correlated flatland theory is described by a theory with weak gravity.
- Conversely strongly interacting gravitational physics can be described by an almost free theory in flatland.

Implications of holography

1. **Tool**: Strongly correlated theories can be solved using weakly interacting gravity in one higher dimension!

2. **Fundamental paradigm**: Quantum gravity emerges from conceptually well-understood field theories akin to QCD, with no gravity.

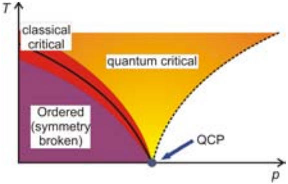
Outline

- A brief introduction to holography
- **Holography as a tool for strongly correlated systems**

Modeling quantum critical points

- Much recent interest in engineering geometries to model quantum critical points.

Find geometries with the right properties to model....



(non)-Fermi liquids, (non) relativistic scale invariant systems, cold atoms, high T_c superconductivity....

Non-relativistic holography

Here we will focus on recent progress in non-relativistic holography.

References

Based on:

- M. Guica, K. Skenderis, M. Taylor, B. van Rees
  Holography for Schrödinger backgrounds,
  1008.1991
- R. Caldeira-Costa and M. Taylor
  Holography for chiral scale-invariant models
  1010.4800

but we will put these results into a wider context.

Why non-relativistic holography?

- Gauge/gravity dualities have become an important new tool in extracting strong coupling physics.
- Strongly coupled non-relativistic QFTs are common place in condensed matter physics and elsewhere.
- It is natural to wonder whether holography can be used to obtain new results about such non-relativistic strongly interacting systems.

Experimental motivations?

1. **Cold atoms at unitarity.**
   - Fermions in three spatial dimensions with interactions fine-tuned so that the *s*-wave scattering saturates the unitarity bound.
   - This system has been realized in the lab using trapped cold atoms. [O'Hara et al (2002) ...]
   - It has been modeled theoretically by Schrödinger invariant theories. [Son et al]
2. **High T_c superconductivity.**
   - Non-relativistic QCP underlying strange metal phases?

Holographic realization

How can a non-relativistic field theory be modeled holographically with a dual spacetime?

Holographic realization

Symmetries of the field theory should be realized holographically as isometries of the dual spacetimes.

Anti-de Sitter in $(D+1)$ dimensions admits as an isometry group the D-dimensional conformal group $SO(D,2)$.

Symmetries of a non-relativistic theory

In D spacetime dimensions the Galilean group consists of:

- temporal translations $\mathcal{H}$, spatial translations $\mathcal{P}^i$, rotations $\mathcal{M}^{ij}$ and Galilean boosts $\mathcal{K}^i$.

The Galilean algebra admits a central extension:

$$[K_i, P_j] = M\delta_{ij},$$

where M is the non-relativistic mass (or particle number).

Schrödinger symmetry group

The conformal extension adds to these generators:

- a dilation generator $\mathcal{D}_2$ and a special conformal generator $\mathcal{C}$.
- The dilatation symmetry $\mathcal{D}_2$ acts as

$$t \rightarrow \lambda^2 t, \qquad x^i \rightarrow \lambda x^i,$$

  i.e. with dynamical exponent $z = 2$.
- This is the maximal kinematical symmetry group of the free Schrödinger equation [Niederer (1972)], hence its name: Schrödinger group Sch_D.

Schrödinger with general exponent z

- One can also add to the Galilean generators (including the mass $\mathcal{M}$) a generator of dilatations $\mathcal{D}_z$ acting as

$$t \to \lambda^z t, \qquad x^i \to \lambda x^i$$

  but for general z there is no special conformal symmetry.
- This algebra will be denoted as $Sch_D(z)$.
- Removing the central term $\mathcal{M}$ gives the symmetries of a D-dimensional Lifshitz theory with exponent z, denoted $Lif_D(z)$.

Lifshitz spacetimes

The Lifshitz symmetry $Lif_D(z)$ may be realized geometrically in $(D+1)$ dimensions [Kachru et al, 2008]

$$ds^2 = \frac{dr^2}{r^2} - \frac{dt^2}{r^{2z}} + \frac{dx^i dx_i}{r^2}.$$

- As in AdS, the radial direction is associated with scale transformations: $r \to \lambda r$, $t \to \lambda^z t$, $x^i \to \lambda x^i$ is an isometry.

Holography for Schrödinger

[Son (2008)] and [K. Balasubramanian, McGreevy (2008)] initiated a discussion of holography for $(D+2)$ dimensional Schrödinger spacetimes,

$$ds^2 = -\frac{b^2 du^2}{r^4} + \frac{2dudv + dx^i dx^i + dr^2}{r^2},$$

- This metric realizes geometrically the Schrödinger group with $z = 2$ in D dimensions: the radial direction is associated with dilatations, whilst another extra direction v is needed to realize the mass operator $\mathcal{M}$.
- In order for the mass operator $\mathcal{M}$ to have discrete eigenvalues the lightcone coordinate v must be compactified, giving a D-dimensional field theory with u the time coordinate.

Holography for general z Schrödinger

More generally one can also realize $Sch_D(z)$ geometrically in $(D+2)$ dimensions via

$$ds^2 = \frac{\sigma^2 du^2}{r^{2z}} + \frac{2dudv + dx^i dx^i + dr^2}{r^2},$$

- The dual field theory is then $(D+1)$-dimensional, with anisotropic scale invariance $u \to \lambda^z u$, $v \to \lambda^{2-z} v$ and $x^i \to \lambda x^i$.
- The theory becomes a non-relativistic theory in D dimensions upon compactifying v or u.

Basic questions about non-relativistic holography

What kind of strongly interacting Lifshitz and Schrödinger invariant theories can holography describe?

1. Matching of phenomenological models to strange metal behavior near superconducting phases?
2. Candidate descriptions for cold atom systems?

Non-relativistic holography

1. **Features of Schrödinger holography**
2. Lifshitz holography

Phenomenological models for Schrödinger

The $(D+2)$-dimensional Schrödinger spacetimes solve the field equations for Einstein gravity coupled to various types of matter. Simplest example [Son, 2008]:

- **Massive vector** model.

$$S = \int d^{D+2}x \sqrt{-G}[R - 2\Lambda - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m^2 A_\mu A^\mu]$$

with $m^2 = z(D + z - 1)$. Schrödinger metric supported by vector field with only a null component:

$$A_u = \frac{b}{r^z}.$$

Field theories dual to Schrödinger

- In general, the field theories dual to $(D+2)$-dimensional Schrödinger geometries [M.T. et al, 2010]

$$ds^2 = \frac{\sigma^2 du^2}{r^{2z}} + \frac{2 du dv + dx^i dx^i + dr^2}{r^2},$$

can all be understood as Lorentz symmetry breaking deformations of $(D+1)$-dimensional CFTs e.g.

$$S_{CFT} \rightarrow S_{CFT} + \int du dv dx^i b X_v + \cdots$$

- Here X_v is a component of a vector operator, with relativistic dimension $(D+z)$.

Exactly marginal deformations

- For $z > 1$ this is an irrelevant deformation of the $(D+1)$-dimensional CFT, whilst for $z < 1$ it is relevant:

$$S_{CFT} \rightarrow S_{CFT} + \int dudvdx^i bX_v + \cdots$$

Such deformations explicitly break the relativistic and conformal symmetry but are **exactly marginal** with respect to $Sch_D(z)$ symmetry.

Schrödinger phenomenology

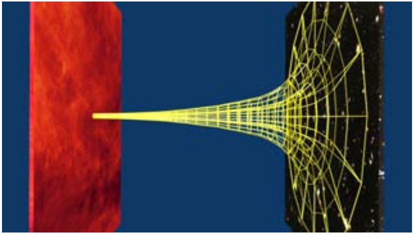
- In the bulk geometries the deformation parameter b can take any value.

The physical systems being modeled should have a corresponding parameter, adjusting which preserves the quantum criticality.

- How can we stabilize this modulus?

Schrödinger phenomenology

Recall that from the perspective of the original CFT, the deformation was irrelevant.



- The IR behavior is dominated by that of the original CFT, whilst the UV behavior is that of a $z = 2$ theory.
- Using probe branes to compute conductivities of charge carriers in the dual theory, one indeed sees such behavior. [Ammon et al, 2010]

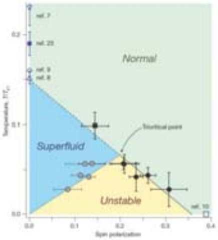
Discrete Lightcone Quantization (DLCQ)

- To obtain a non-relativistic system we still need to compactify a null direction.

But periodically identifying a null circle is subtle!

- The zero mode sector is usually problematic (and here the problem is seen in ambiguities in the initial value problem in the spacetime).
- Strings winding the null circle become very light.

Schrödinger critical points



(Y. Shin et al, Nature (2008))

- The Schrödinger geometries were introduced to model cold atom systems.
- However, holographic models correspond to non-relativistic deformations of a CFT in one null dimension higher: their phenomenology looks quite different!

Schrödinger summary

Geometric realization of the particle number $\mathcal{M}$ in the Schrödinger algebras using a null direction is undesirable (viewed from perspective of cold atom physics).

Schrödinger critical points

Sometimes one doesn't get quite what one expected, but this is not always a bad thing...

Schrödinger critical points

The Schrödinger theories could be understood in terms of deformations of relativistic conformal field theories

$$\mathcal{L}_{\rm CFT} \rightarrow \mathcal{L}_{\rm CFT} + b\mathcal{X},$$

where $\mathcal{X}$ is an operator that breaks Lorentz invariance.

Generic choices of the operator $\mathcal{X}$ break relativistic scale invariance, but exactly preserve Schrödinger invariance for any parameter b.

Non-relativistic holography

1. Schrödinger holography
2. **Features of Lifshitz holography**

Phenomenological models for Lifshitz

The $(D+1)$-dimensional Lifshitz spacetimes solve the field equations for Einstein gravity coupled to various types of matter. Simplest example [M.T., 2008]:

- **Massive vector** model.

$$S = \int d^{D+1}x \sqrt{-G}[R - 2\Lambda - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m^2 A_\mu A^\mu]$$

with $m^2 = z(D+z-2)$. Lifshitz metrics are supported by a vector field with only a timelike component:

$$A_t = \frac{1}{r^z}.$$

Relation to Schrödinger

- Embedding Lifshitz into string theory, the best understood examples [Donos, Gauntlett] turn out to be DLCQs of $z = 0$ Schrödinger in one higher dimension [M.T. et al] :

$$ds^2 = \frac{dr^2}{r^2} + \sigma^2 du^2 + \frac{1}{r^2}(2dudv + dx^i dx_i)$$

Reducing over the u coordinate gives 4d Lifshitz with $z = 2$:

$$ds^2 = \frac{dr^2}{r^2} + \sigma^2(du + \frac{dv}{\sigma^2 r^2})^2 - \frac{dv^2}{\sigma^2 r^4} + \frac{dx^i dx_i}{r^2}.$$

Lifshitz holography

The Lifshitz field theory again follows from a DLCQ of a deformation of a CFT which breaks relativistic conformal symmetry but preserves Lifshitz symmetry.

Lifshitz holography

1. What is the physical role of the deforming operator in the quantum critical theory?
2. Are **all** holographic realizations of Lifshitz actually deformations of relativistic field theories?

Lifshitz phenomenology

- Consider an action which includes a gauge field and a scalar [M.T. 2008] :

$$S = \int d^{D+1}x \sqrt{-G}[R - \frac{1}{2}(\partial\Phi)^2 + g(\Phi)F_{\mu\nu}F^{\mu\nu} + V(\Phi)]$$

  These actions admit Lifshitz black hole solutions

$$ds^2 \sim -f(r)r^{\beta}dt^2 + \frac{dr^2}{r^{\beta}f(r)} + r^{\gamma}dx^i dx_i,$$

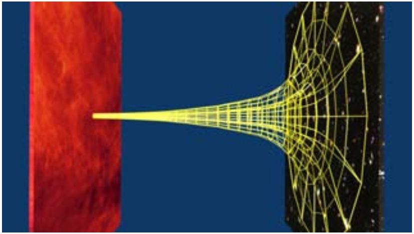
  with $f(r) = 0$ at the horizon, and $f(r) = 1$ in zero temperature solutions. (Zero entropy extremal BH!)
- The metric is Lifshitz at $T = 0$, but the field equations enforce a running scalar

$$\Phi \sim \log(r),$$

  which breaks the scale invariance.

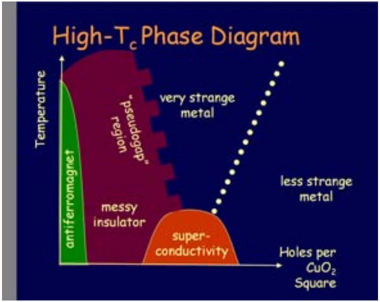
Lifshitz and strange metals

Probe branes in Lifshitz can model charge carriers interacting with the quantum critical theory:



- The charge carriers have DC resistivity $\rho \sim T^{v_1}$ and AC conductivity behaves as $\sigma(\omega) \sim \omega^{-v_2}$, with nontrivial v_1 and v_2. [Hartnoll et al, 2009]
- Strange metal behavior requires $v_1 \sim 1$ and $v_2 \sim 0.6$.

Strange metals and superconductors



(TCM, Cambridge)

- This fits with proposals of an underlying non-relativistic critical point.
- Currently different geometries can model holographically parts of the phase diagram.
- The key challenge is to find a geometry capturing all of the diagram at once.

Holographic superconductors

- The superconducting phase (at finite temperature) is modeled by black hole hair.
- Usually "black holes have no hair" so this phase is rather novel.
- The hair characterizes the long range order in the superconductor.

(NASA)

Lifshitz outlook

Modified Lifshitz black holes may model strange metal behavior, but the picture needs to be developed further, connecting to superconducting and other phases.

Summary

- **Conclusions and outlook**

Non-relativistic holography: status

General success:

- Simple phenomenological models capture key features of strongly interacting non-relativistic theories.

Open problem:

- Neither Lifshitz nor Schrödinger has been satisfactorily embedded into top-down string theory models, and many holographic calculations are technically and conceptually challenging.

Schrödinger and Lifshitz critical points

We discovered that the holographic Schrödinger and Lifshitz theories could be understood in terms of deformations of relativistic conformal field theories

$$\mathcal{L}_{\rm CFT} \to \mathcal{L}_{\rm CFT} + b\mathcal{X},$$

where $\mathcal{X}$ is an operator that breaks Lorentz invariance.

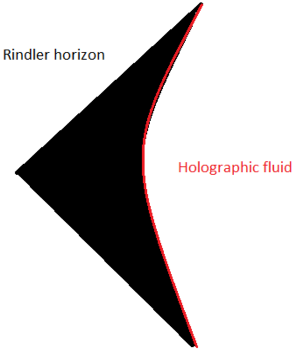
The operator $\mathcal{X}$ is chosen to break relativistic scale invariance, but preserve Schrödinger or Lifshitz symmetry for any parameter b.

Non-relativistic critical points

- This procedure generates in a controlled way a wide range of non-relativistic QCP, generalizing [Cardy, 1991].
- It does not rely on a holographic realization, although we can use holography to realize strongly interacting models.

Non-relativistic critical points

Given gravity in the bulk is relativistic, it is unsurprising that the non-relativistic field theories turned out to be deformations of relativistic theories.



- A correspondence between a non-relativistic holographic fluid and vacuum Einstein solutions has recently been proposed [Bredberg et al].
- The non-relativistic fluid can also be understood in terms of a relativistic theory with spontaneous (Lorentz) symmetry breaking. [Compère et al].

Relativistic or non-relativistic?

Will all holographic realizations of **non-relativistic theories using Einstein gravity** inevitably turn out to be **relativistic** theories in which the Lorentz symmetry is spontaneously and/or explicitly broken?

Non-relativistic bulk theory?

- Using relativistic (Einstein) gravity to model a non-relativistic field theory is perhaps counter-intuitive.

Should one instead take a non-relativistic limit of AdS/CFT, in which the bulk theory is Newtonian? [Bagchi, Gopakumar]

- No black holes?

Challenges and lessons from holography

There are other rather generic issues in holographic modeling:

- Validity of classical gravity in the bulk requires large N limit of CMT system.

So what should we hope ultimately to learn from holography?

- We may obtain novel effective Hamiltonian descriptions, emerging naturally from holographic systems.

Future prospects?

Holographic models look promising for capturing phenomena from a wide range of strongly interacting systems.