# Contents

Notations iii  
Abstract v  
Introduction vii  

## 1 Brans-Dicke Theory. 1  
  1.1 Machs Principle. 1  
  1.2 The Theory 3  
    1.2.1 Positioning in Scalar-Tensor theories of gravity 5  
    1.2.2 Gravitational waves 6  
  1.3 Limit of General Relativity 8  
  1.4 Experimental Bounds 9  
    1.4.1 Several experiments 10  
  1.5 Cosmological features in Brans-Dicke theory 13  
    1.5.1 Extended Inflation 13  
    1.5.2 Density perturbations 16  
    1.5.3 Dark energy 17  

## 2 Baryogenesis. 19  
  2.1 General Outline 19  
  2.2 Shakarov Conditions 20  
  2.3 GUT Baryogenesis 21  
    2.3.1 Short introduction to SU(5) GUT 21  
    2.3.2 Preheating 23  
  2.4 Electroweak Baryogenesis 24  
    2.4.1 Sphaleron processes 24  
    2.4.2 CP violation in extended models 25  
    2.4.3 First order phase transition 26  
  2.5 Affleck-Dine Mechanism 28  
    2.5.1 The mechanism in a toy model 29  
    2.5.2 AD mechanism with supersymmetry 30  
  2.6 Leptogenesis 32  
    2.6.1 See-saw mechanism 32
## CONTENTS

2.6.2 Lepton number generation .................................. 33  
2.7 Gravitational baryogenesis .................................... 34  

### 3 Scalar fields .................................................. 37  
3.1 Inflation .......................................................... 39  
3.2 CP violation ........................................................ 41  

### 4 Baryogenesis in Brans-Dicke theory ......................... 43  
4.1 Introduction ...................................................... 43  
4.2 Generalization of the theory ................................... 44  
4.3 Double kinetic term ............................................. 46  
  4.3.1 Powerlaw expansion ......................................... 50  
  4.3.2 Incorporation in extended inflation ....................... 52  
4.4 Higher order terms .............................................. 54  
  4.4.1 Cubic term .................................................. 55  
  4.4.2 Quartic term ................................................ 56  
4.5 Massless minimal coupling ..................................... 58  
  4.5.1 Cubic term .................................................. 58  
  4.5.2 Quartic term ................................................ 61  
4.6 Fate of the condensate ......................................... 65  

### 5 Conclusion ...................................................... 67  

### A Short introduction to super symmetry ....................... 69  
A.1 Supersymmetric Lagrangian .................................... 71  
A.2 MSSM ............................................................ 72  
A.3 Flat directions .................................................. 73  

Acknowledgements .................................................. 77  

Bibliography ......................................................... 78
Notations

Greek indices run from 0 to 3 with the first the time coordinate.

Conformal time is defined as \( a(\eta)d\eta = dt \).

The scale factor is denoted by \( a(t) \) or \( a(\eta) \) in physical and conformal time respectively.

Derivation with respect to physical time is denoted by a dot, derivation with respect to conformal time by a prime.

Unless stated otherwise, the spacetime metric is the one of de Sitter space denoted by \( g_{\mu\nu} = \text{diag}(-1, a^2, a^2, a^2) \) in physical time. The determinant of the metric is denoted by \( g \).

The Hubble parameter \( H \) is given by \( H = \dot{a}/a \). The Ricci curvature tensor is written \( R_{\mu\nu} \) and the Ricci scalar \( R = g^{\mu\nu}R_{\mu\nu} \). We will use that during inflation the Ricci scalar is given by \( R = 12H^2 \).

The d’Alembertian \( \Box \) is defined by \( \Box = (-g)^{-1/2}\partial^\mu(\sqrt{-g}\partial_\mu) \). In de Sitter space, most used in this thesis, this yields \( \Box = a^{-3}\partial^\mu(a^3\partial_\mu) \).

The complex conjugate of scalar fields is denoted by *, charge conjugation of spinors by the superscript \( ^c \). When the complex conjugated term in a lagrangian are added this is simply written by +c.c..

A bar denotes the adjoint spinor \( \bar{\psi} = \psi^\dagger \gamma^0 \), where the dagger denotes the hermitian and \( \gamma^0 \) denotes the zeroth order gamma matrix in the Pauli-Dirac representation.

The real and imaginary part of coefficients and fields are denoted by the subscripts \( r \) and \( i \) respectively. Complex fields like \( \phi \) are often decomposed in their real and imaginary part \( \phi = \phi_r + i\phi_i \) and written in the vectorial form \( \phi_i = \begin{pmatrix} \phi_r \\ \phi_i \end{pmatrix} \). A rotated field is decomposed in the \( \phi_\pm \) fields such that
$\phi = \phi_+ + i \phi_-$ and $\phi_\pm = M(\theta) \phi_\mp$ with $M(\theta)$ the O(1) rotation matrix that rotates over an angel $\theta$. All fields $\phi_\pm$ and $\phi_\mp$ are real.

The Noether current associated with a conserved quantity of a complex field $\phi$ is denoted by $J_\phi^\mu$. We are often only interested in the zeroth component written as $J_\phi^0 = J_\phi = n_B$, in which $B$ corresponds to the associated quantity.
Abstract

After an introduction to Brans-Dicke theory and a review of several established theories of baryogenesis a new possible mechanism is considered in this thesis. The scalar field of Brans-Dicke theory is generalized to a complex field with nontrivial CP violating potential. First it is shown that a combination of kinetic terms result in CP violation on tree level, enhancing the homogeneous modes and thus producing a net baryon current. This model can in an elegant way be incorporated into a more extended model of inflation. Secondly, a model with a cannonical kinetic term is investigated. A mean field approximation is made such that the interaction terms can be simplified to solve the equation of motion analytically. It is assumed the process of baryogenesis takes place during inflation at the time the scalar can effectively be treated as massless minimally coupled, since it is argued a massive minimally coupled scalar can not give any production. It is shown that homogeneous modes can get a growing solution when including cubic or quartic terms, but nonzero modes cannot be enhanced. The solutions depend in a variety of ways on initial conditions.
Introduction

In 1928 Paul Dirac formulated the famous Dirac equations and predicted the existence of anti-particles. It was only 4 years later when Carl Anderson experimentally proved the existence of the positron. Since anti-matter has exactly the same mass and opposite charge, the question soon arose why only matter is seen in the universe - all symmetries in the laws of nature seemed not to favor one over the other. It was possible to simply postulate the existence of the asymmetry, making it an initial condition of the universe. But besides the fact that physicists do not normally settle for initial conditions of this kind, Alan Guth’s inflation (1979) spoiled this possibility. All possible excitations are supposed to dilute in the exponential expansion phase in the very early universe. But the quest for baryogenesis was started already in 1967, when Sakharov published three conditions a model has to obey in order to push the universe to an asymmetric state of matter. An important role plays the presence of CP violation, the second condition of Sakharov. In 1950 the symmetry of the combination of charge conjugation (C) and parity (P) was postulated when it was discovered that P was not a symmetry of nature. The hope for a new symmetry was destroyed in 1964 when Cronin and Fitch discovered the CP violating decay of neutral kaons, for which they won the Nobel price in 1980. The first serious model of baryogenesis relied on GUT’s, Grand Unification Theories, first proposed in the late 70’s. Since then many models and mechanisms are considered in an enormous detail but still no consensus is reached about the ultimate theory. The CP violation present in many theories is often too small to reach a large enough asymmetry. Or departure from thermal equilibrium, the third condition, can only be reached with bounds on particle masses that are under siege by the constantly improving particle accelerators. This leaves the door open for research on new ideas and perhaps new physics beyond the established theories.

Brans-Dicke theory is not exactly to be called new physics. It was proposed by C. Brans and R.H. Dicke already in 1961 as a theory of gravity according to Machs principle. General Relativity is not compatible with this notion of space and the two scientists realized that including the variability of the gravitational constant $G$ would make GR 'Machian'. This resulted
in the introduction of the Brans-Dicke scalar, responsible for the dynamics of the not anymore constant $G$. More research was done on the topic and interesting cosmological models were build on it, but as time went by and experiments constrained the theory, some of the interest was lost. Recently it experienced a revival when it was realized that string theory inevitable contained a scalar field, the dilaton, much alike the Brans-Dicke field. The dilaton also mediates gravitational effects and has almost the same action. In combination with the fact that generalizations of the original theory, dubbed Scalar-Tensor theories of gravity, are still far from exclusion, it seems worth while to investigate the possibilities of baryogenesis in this theory.

As mentioned before, current observations have restricted the theory in a serious way, but small complexifications of the original theory can solve these problems. For simplicity we will consider the original Brans-Dicke action in this thesis and extend it to a complex scalar with the addition of a CP violating potential. The elegance of this model is that CP violation can directly be combined with a theory of gravity, something which was not possible in General Relativity. It seems very natural to have the same theory responsible for the gravitational interactions of matter as well as the production of matter in the early universe. It will also turn out to be effective during the inflationary epoch and perhaps also responsible for the rapid expansion in the same era.

The thesis is organized in the following way. We will first give an introduction to Brans-Dicke theory and point out some of its features. The next chapter explains the basic problems and aspects of baryogenesis and treats the most accepted models under consideration in a concise manner. After a short piece on complex scalar fields we will consider a new model for baryogenesis based on Brans-Dicke theory. An introduction to supersymmetry is added in the appendix, since this theory is widely used in modern models of baryogenesis.
Chapter 1

Brans-Dicke Theory.

1.1 Mach’s Principle.

The true nature of space and time has already been a point of discussion for a long time. Some people like Descartes proposed space is a structure on its own, having intrinsic properties and in that sense an independent concept. Newton embraced this idea of absolute space in his ‘Principia Mathematica Philosophiae Naturalis’ and also in later times it was favoured, for instance when ether was introduced. On the other hand there was the point of view of people like Bishop Berkeley and Ernst Mach. They proposed that space was meaningless without matter or energy in it. In other words, concepts like geometry and other properties of space are meaningless in an empty universe. The presence of matter is inherent to the contains of the universe. When boundary conditions supporting this idea are imposed in General Relativity, the vacuum solutions break down, so one might say this theory is not compatible with this notion of space and time.

The conflict between Mach’s principal and General Relativity (GR) can be illustrated by a simple (although slightly imperfect) Gedanken experiment. Consider a completely empty universe except for a single laboratory with an experimenter floating around in it. In GR we can now fix a traditional Minkowskian coordinate system to the laboratory and assume the mass of it to be very small. In that case we can take its effect on the metric to be very small and thus use weak field approximation. The experimentalist will find its experiments to obey all usual laws of physics in the common way. But now he fires a bullet tangential away from its laboratory into space. The laboratory will start to rotate and therefore an onboard gyroscope will start to rotate with respect to the walls and the observer. It will be fixed nearly exact to the direction of motion of the bullet. This means that the laws of physics treat a fast receding bullet of a very small mass with more importance than the very close walls of exceeding mass. This sense of space has more in common with Newton’s absolute space than it has with
CHAPTER 1. BRANS-DICKE THEORY.

physical space as Mach described. Assuming we would like to embrace the point of view of space having no intrinsic geometrical properties except of those coming from the matter contained in it and GR does not have ‘hidden’ boundary conditions, there is only one option left. This situation cannot be described correctly by the laws of GR. Brans-Dicke theory as described by C. Brans and R.H. Dicke in their 1961 paper [1] gives a possible solution to this problem by proposing an alternative theory of gravity. They based their assumptions on work of Jordan [2], who constructed together with Fierz the frame in which the theory is set. Therefore the theory is sometimes also referred to as the Jordan-Fierz-Brans-Dicke (JFBD) theory.

Taking Mach’s principle to be valid and from dimensional argument we can make the following assumptions. Let $M$ be the mass content of a sphere which is in causal contact and let $R$ be its radius. These two can then be related like,

$$\frac{GM}{Rc^2} \sim 1,$$

(1.1)

where $G$ denotes Newton’s gravitational constant and $c$ the speed of light. To get a physical feeling for this relation, consider a laboratory near a star. Besides the gravitational pull of the star the observer experiences an inertial reaction. According to Mach’s principle, however, motions are meaningful only when considered relative to the rest frame of all the masses in the universe. We therefore can imagine a very distant accelerating mass creating an equivalent force at the observer which coincides with this inertial reaction. In the frame of a free falling observer the gravitational force of the star is precisely cancelled by the inertial force. When these two forces are multiplied by the same factor, nothing observable happens from the point of view of the laboratory. This means the acceleration is determined by the distribution of mass in space but is independent on the strength of the gravitational interaction. Denoting the mass of the sun by $M_\odot$ and the distance to it by $r$, we can write the acceleration due to gravitation according to Newton by $a = GM_\odot/r^2$. The acceleration due to the equivalent inertial force should then be given by $a \sim M_\odot R c^2 / M r^2$ and combining the two gives equation (1.1). This is a rather qualitative way of writing, but anyhow this result suggests two possibilities. Either the ratio $M/R$ should be fixed in a reasonable theory or the gravitational constant should locally be variable and determined by the distribution of mass in the universe. The first option was what people hoped to find as a result of boundary conditions on the field equations of General Relativity. The second is incompatible with GR, since it violates the strong equivalence principle on which GR is build. The strong equivalence principle states that the laws of physics including dimensionless physical constants are equal for all space-time points and thus excludes variation of the gravitational constant. However, only the weak equivalence principle, which states that all gravitational accelerations are locally equal, is experimentally supported (Eötvös). This does not conflict
with a variable gravitational constant and from now on we will assume this is indeed the case. Every piece of matter in space should in some sense contribute to the reciprocal of the gravitational constant if in causal contact. In a symbolic way this is written like,

\[ G^{-1} \sim \sum_i \frac{m_i}{r_i c^2}. \] (1.2)

With this notion a theory of gravity is formulated which is compatible with Mach's principle and at the same time should converge to General Relativity in a certain limit.

1.2 The Theory.

First of all, this theory is a generalization of General Relativity and not a complete theory on its own. Gravitational effects are described not only by the metric but by a scalar field as well. The scalar is set in a Riemannian manifold in which gravity is partly due to scalar interactions and partly geometrical. Varying the gravitational "constant" \( G \) means introducing some scalar field that carries this variation. No known scalars in GR are suitable for this purpose, as the contracted metric tensor is constant and all scalars formed from the curvature tensor fall off more rapidly than \( r^{-1} \) from the mass source. This means we will have to introduce a new scalar field, which we will denote by \( \Phi \). Since we want to generalize General Relativity instead of building a complete new theory, we start with the Hilbert-Einstein action that gives rise to the equations of motion of matter and non-gravitational fields in GR,

\[ S = \frac{c^4}{16\pi G_0} \int d^4x (-g)^{1/2} \left[ R + \frac{16\pi G}{c^4} L \right]. \] (1.3)

Here we denote by \( R \) the Ricci curvature scalar, by \( g \) the determinant of the metric \( g_{\mu\nu} \) and by \( L \) the Lagrangian of matter and non-gravitational fields. The metric is chosen to have the signature \( \text{Diag}(-1,1,1,1) \). To generalize this action and introduce the variability of \( G \), we include a lagrangian density of the scalar field \( \Phi \) and assume \( G \) to be a function of this field. Looking at Eq. (1.2) it would make sense to assume \( G^{-1} \) to vary as \( \Phi \). In this way, a wave equation for \( \Phi \) sourced by the matter distribution of the universe gives qualitative the same relation as Eq. (1.2). We get,

\[ S = \frac{c^4}{16\pi G_0} \int d^4x (-g)^{1/2} \left[ \Phi R - \frac{\omega}{\Phi} (\partial_\mu \Phi \partial^\mu \Phi) g^{\mu\nu} \right] + \int d^4x (-g)^{1/2} L. \] (1.4)

To make sure the coupling constant \( \omega \) of the kinetic term of \( \Phi \) is dimensionless, it is divided by the scalar. \( G_0 \) is a dimensionless number such that \( G = G_0/\Phi_0 \) is the current observed constant of gravity. If \( G_0 = 1 \),
Φ₀ = M₂. It should be noted that the extra terms only affect the gravitational field equations which determine the metric. The equations of motion resulting from varying this action with respect to matter fields will not be any different of those derived in General Relativity. Therefore a particle will move the same way in a given metric in both theories. Now we can perform all the usual tricks on this action to examine its properties. We start by writing down the wave equation for Φ using the variational principal,

\[ 2ωΦ^{-1}\Box Φ + \frac{ω}{Φ²}\partial^μΦ\partial_μΦ + R = 0. \]  

(1.5)

The curvature clearly acts as a source for the generation of waves in Φ. We will rewrite this equation in a moment in a much more attractive way, but before that we first need to know the analog of the Einstein field equations. These can be derived by taking the variation of the action with respect to the metric,

\[ R_{μν} - \frac{1}{2}g_{μν}R = \frac{8πG₀}{Φc^4}T_{μν} + \frac{ω}{Φ²}(\partial_μΦ\partial_νΦ - \frac{1}{2}g_{μν}(\partial^ρΦ)(\partial_ρΦ)) + \frac{1}{Φ}(∇_ν(\partial_μΦ) - g_{μν}\Box Φ). \]  

(1.6)

The left hand side looks familiar, being the usual Einstein tensor. The first term on the right hand side is familiar as well, except for the substitution of the gravitational constant by the scalar field. The second term is the energy-momentum tensor of the scalar field, similar to the first term and also with the Φ⁻¹ coupling 'constant'. The last term is unusual and stems from the second derivatives that result from taking the variation. These second derivatives can be eliminated by partial integration, resulting in a total divergence and the extra term. With the proper boundary conditions, the total derivative vanishes. It will be shown in a moment that this term is an important addition to the Einstein equations. As in General Relativity, any viable theory should include conservation of energy-momentum, expressed by,

\[ ∇_νT^{μν} = 0 \]  

(1.7)

To be sure Eq. (1.7) will be consistent with Eqs. (1.5) and (1.6), this term is definitely needed. A beautiful property of the wave function Eq. (1.5) becomes clear when we take the trace of Eq. (1.6),

\[ -R = \frac{8πG₀}{Φc^4}T + \frac{ω}{Φ²}(\partial^μΦ)(\partial_μΦ) - \frac{3}{Φ}\Box Φ, \]  

(1.8)

and substitute this result in Eq. (1.5) to get the following wave equation for Φ,

\[ \Box Φ = \frac{8πG₀}{(3 + 2ω)c^4}T. \]  

(1.9)
We now see that the trace of the energy-momentum tensor of matter sources waves in the scalar field. This is completely in correspondence with Mach's principle saying that $\Phi$ should be sourced by the matter distribution of the universe.

1.2.1 Positioning in Scalar-Tensor theories of gravity

Brans-Dicke theory is in fact a special case of more general Scalar-Tensor theories of gravity. The foundations were made by the attempts of Kaluza and Klein to unify General Relativity with the theory of electromagnetism into a 5 dimensional geometric theory. In such a theory gravity is naturally described by a spin 2 tensor and a spin zero scalar field called the dilaton. The remaining degrees of freedom are occupied by the electromagnetic four-vector and this is therefore a Scalar-Vector-Tensor theory. The fifth dimension was supposed to be wrapped up, compactified, in order to explain its absence in daily life. In string theories the dilaton appears as well as a scalar associated with gravity. The action of the dilaton resembles that of Brans-Dicke theory with $\omega = 1$ and the field substitution $\Phi = \exp(-2\phi)$.

In the most simple form, a tensor-scalar theory has a coupling constant $\omega$ depending on the field. The action is in such case given by,

$$S_J = \frac{c^4}{16\pi G_0} \int d^4 x (-g)^{1/2} \left[ \Phi R + \frac{\omega(\Phi)}{\Phi} (\partial_{\mu}\Phi \partial_{\nu}\Phi) g^{\mu\nu} \right] + \int d^4 x (-g)^{1/2} L_m .$$

Here $L_m$ denotes the lagrangian of matter and non-gravitational fields $\Psi_m$ and from now on we will set $c = G_0 = 1$. The wave equation for $\Phi$ is now extended in comparison with eqn. (1.9) to,

$$\Box \Phi = \frac{1}{3 + 2\omega(\Phi)} (8\pi T - \omega'(\Phi)(\partial_{\mu}\Phi \partial_{\nu}\Phi) g^{\mu\nu}) .$$

The prime denotes derivation with respect to the field. Using a conformal transformation $g_{\mu\nu} \rightarrow A^2(\Phi) \tilde{g}_{\mu\nu}$, the action can be written in such a way that the field equations take the same form as in GR, with the addition of the scalar field $\psi$. The frame corresponding to the metric $\tilde{g}_{\mu\nu}$ is called the Einstein frame, while eqn. (1.10) is written in the Jordan frame with metric $g_{\mu\nu}$. The Einstein action then reads,

$$S_E = \int d^4 x (-\tilde{g})^{1/2} \left( \frac{1}{16\pi} \tilde{R} + \frac{1}{2} (\partial_{\mu}\psi \partial_{\nu}\psi) \tilde{g}^{\mu\nu} \right) + S_m [\Psi_m, A^2(\Phi) \tilde{g}^{\mu\nu}] .$$

The coupling to curvature has disappeared and instead the scalar couples independently to all other ordinary matter and radiation fields in the matter lagrangian. It depends on what kind of problem is to be solved when one has to decide which frame is most suitable to use. From the action (1.12)
we can derive the field equations and the wave equation of the scalar by extremizing the action with respect to $\bar{g}_{\mu\nu}$ and $\psi$ respectively,

$$\bar{R}_{\mu\nu} = -8\pi(\bar{T}_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}\partial_\mu\psi\partial_\nu\psi)$$  \hspace{1cm} (1.13a)

$$\Box\psi = -\sqrt{4\pi\alpha}\bar{T}.$$  \hspace{1cm} (1.13b)

Here we define $\alpha^{-2} = 2\omega(\Phi) + 3$, $\sqrt{4\pi\alpha} = \partial\Gamma/\partial\psi$, $\Gamma = \log A(\Phi)$ and the stress-energy tensor in the Einstein frame with respect to $\bar{g}_{\mu\nu}$ is $\bar{T}_{\mu\nu} = A^6T_{\mu\nu}$.

Since only in the Jordan frame the stress energy tensor is conserved, we have to consider this frame to be the physical if one would assure energy conservation. One of the benefits of this more general theory is the fact that the coupling can run. As will be explained in section 1.4, experimental bounds restrict $\omega$ to be quite strict, contradicting to the demands of many interesting cosmological models. With $\omega$ depending on the field, one can construct the situation with a small $\omega$ and large scalar during the early universe, while the constraints of todays observation are still met.

### 1.2.2 Gravitational waves

Similar to General Relativity, gravitational waves occur in Brans-Dicke theory and are an important feature to discuss. In addition to the familiar spin two tensor particles we have now to deal with spin zero scalar particles as well. This influences the energy radiated by mass systems, as will be explained in section 1.4. For convenience we will work in the Einstein frame as explained in the previous section. We start with the action stated in eqn. (1.12) in the limit of Brans-Dicke theory. In order to do so we have to make the choice for $A(\Phi)$ to be,

$$A^2(\Phi) = \frac{1}{\Phi} \propto \exp\left[-\psi\left(\frac{16\pi}{3 + 2\omega}\right)^{1/2}\right], \quad \omega = \text{const}.$$  \hspace{1cm} (1.14)

The wave equations for the tensor field and the scalar field are then given by,

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi T_{\mu\nu} + 8\pi(\partial_\mu\psi\partial_\nu\psi - \frac{1}{2}g_{\mu\nu}\partial_\rho\psi\partial^\rho\psi)$$  \hspace{1cm} (1.15a)

$$\Box\psi = \left(\frac{4\pi}{3 + 2\omega}\right)^{1/2}T.$$  \hspace{1cm} (1.15b)

The d’Alembertian $\Box$ is defined in the Einstein frame as well as the Ricci tensor and scalar and the stress-energy tensor. The last two terms on the right hand side correspond to the contribution of the scalar to the stress-energy tensor. From eqn. (1.15a) it is clear that the vacuum solutions of this theory equal those of General Relativity when $\psi$ is constant, like it should.
1.2. THE THEORY.

We can now apply the usual weak field approximation to linearize the wave equations,

\[ g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad \text{and} \quad \psi = \psi_0 + \tilde{\psi}. \]  

(1.16)

Here are \( h_{\mu\nu} \) and \( \tilde{\psi} \) small perturbations, \( \eta_{\mu\nu} \) denotes the metric of flat Minkowski space and \( \psi_0 \) is constant which we will choose to be zero. To first order perturbation we arrive at,

\[ G^{(1)}_{\mu\nu} = 8\pi T_{\mu\nu}[\eta_{\mu\nu}, \tilde{\psi}] \]  

(1.17a)

\[ \Box^{(0)} \tilde{\psi} = \left( \frac{4\pi}{3 + 2\omega} \right)^{1/2} T[\eta_{\mu\nu}, \tilde{\psi}], \]  

(1.17b)

with the Einstein tensor in the first order perturbation equal to the GR case,

\[ G^{(1)}_{\mu\nu} = \frac{1}{2} \left[ -\partial_\mu \partial_\nu h - \Box^{(0)} h_{\mu\nu} + \partial_\rho \partial^\alpha h_{\alpha\mu} + \partial_\nu \partial^\alpha h_{\rho\alpha} + \right. \]
\[ \left. -\eta_{\mu\nu} \left( -\Box^{(0)} h + \partial^\alpha \partial^\beta h_{\alpha\beta} \right) \right]. \]  

(1.18)

Here the d’Alembertian \( \Box^{(0)} \) is defined with the flat space metric \( \eta_{\mu\nu} \). The stress-energy tensor now also includes the scalar contribution. In the case of the tensor field, the usual machinery for calculating gravitational waves can be performed, yielding the same result as in GR. The difference comes from the fact that, as explained in the previous section, the Jordan frame is the one we have to take as the physical frame. When transforming from the Einstein frame to the Jordan frame, the scalar field re-enters the equations through the stress-energy tensor and the metric.

As a short note we discuss the effect of incorporation of a potential for \( \Phi \), which has not been done in the previous sections. In some models based in BD theory there is a strong demand for a potential to anchor the expectation value at late times such that the restrictions on \( \omega \) are weaker. This changes the solution for the gravitational waves in an interesting way. Consider a potential \( V(\psi) \) in the Einstein frame. The wave equation of the scalar then changes to,

\[ \Box \psi - \frac{dV}{d\psi} = \left( \frac{4\pi}{3 + 2\omega} \right)^{1/2} T. \]  

(1.19)

As a result, there is a possibility of tachyonic motion, known as Cherenkov radiation, for a moving particle associated with the scalar gravitational wave [3]. This happens when the potential has the property,

\[ \frac{d^2V}{d\psi^2}(\psi_0) < 0. \]  

(1.20)

In this case the group velocity will exceed the speed of light. Later on there will be a more extended discussion on scalar gravitational waves and their influence on binary systems.

---

Baryogenesis in Brans-Dicke theory
1.3 Limit of General Relativity.

Now we have set up a theory, the next question is to what extend this theory describes indeed reality. In this light we should have a closer look at the coupling strength $\omega$ of matter to the scalar field. At a first thought one might expect the theory to asymptotically converge to General Relativity when the coupling strength goes to infinity. This means a certain solution to the field equations in Brans-Dicke theory with energy-momentum distribution $T_{\mu\nu}$ should reduce to the solution of the Einstein equation for the same $T_{\mu\nu}$.

At the time Brans-Dicke theory was introduced this was the general belief, mainly because of the great successes of GR as being the theory describing gravity best. In a heuristic way it feels right to consider more extended models in a certain limit in such a way that the old, established theory is retrieved again. Of course a typical example is GR itself, converging to Newton's theory of gravity when considering small masses. When looking at Eq. (1.9) one would suspect the right hand side to vanish in the limit $\omega \to \infty$. That would mean the scalar is no longer sourced by the trace of the energy-momentum tensor and hence the gravitational "constant" is decoupled from the matter content in the universe. In this way GR is reobtained in this limit. To view this in a more formalistic way, consider Eq. (1.9) for large $\omega$.

We can then write \[ \Phi = \langle \Phi \rangle + O(\Phi_0^2) = \frac{1}{G_0} + O\left(\frac{1}{\omega}\right). \] (1.21)

Here denotes $G_0/\Phi_0$ the value of the gravitational constant today. Substituting this result in Eq. (1.6) gives the familiar expression of GR plus terms going to zero when $\omega$ tends to infinity, \[ R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi T_{\mu\nu} + O\left(\frac{1}{\omega}\right). \] (1.22)

However, as pointed out by Romero and Barros [5], the expansion as given in Eq. (1.21) is not generally true. As it turns out, only when the trace of the energy-momentum tensor $T^\mu_{\mu}$ does not vanish the statements above hold. When $T^\mu_{\mu} = 0$, no information can be obtained anymore from Eq. (1.9), as the righthand side vanishes anyway. A rough order of magnitude estimate, supported by several examples of exact solutions [6], can be calculated from Eq. (1.8). Since we know $T^\mu_{\mu} = 0$ and $\Box \Phi = 0$ we can write, \[ \omega(\partial^\mu \log(\Phi))(\partial_\mu \log(\Phi)) = R. \] (1.23)

which yields, in contrast with Eq. (1.21), \[ \Phi \sim \Phi_0 + O\left(\frac{1}{\sqrt{\omega}}\right). \] (1.24)
1.4 Experimental Bounds

Although $\Phi$ in the limit $\omega \to \infty$ will still approach its constant value, the energy-momentum tensor as given in Eq. (1.6) will not turn into the Einstein equations. The last term does not vanish and therefore gives rise to a contribution not present in General Relativity. A similar situation arises when considering a nonzero cosmological constant. The equivalent of Eq. (1.9) then reads,

$$\Box \Phi = \frac{2\Lambda \Phi}{2\omega + 3}.$$

(1.25)

Since the righthand side contains $\Phi$, the previously used expansion is no longer valid. It turns out the correct $\omega$ behavior is again described like the expansion as given in Eq. (1.24). We can conclude Brans-Dicke theory only converges to General Relativity when the trace of the energy-momentum tensor is nonzero and the cosmological constant vanishes.

At first it was supposed to be a natural thing and an expected property of the theory to converge to GR. The fact that it turns out not to be the case certainly does not mean the end of the theory. The difference between the Newtonian limit of General Relativity and the $\omega \to \infty$ lies in the dependent parameter. Newton’s theory and GR both have the gravitational coupling constant $G$ as a common parameter. In Brans-Dicke theory, however, $\omega$ is an ambiguous parameter which has no equivalent in GR. In this sense, Brans-Dicke theory is a different theory of gravitation as a whole, although in a certain extend very similar to GR. From the Machian point of view it can be put like this. We have to make the distinction between local problems and cosmological problems. Locally, we are restricted to the solutions with asymptotic behavior of $\Phi = \Phi_0 + O(1/\omega)$. This represents the presence of matter in the universe defining $G$ in our neighborhood in such a way the metric $g_{\mu \nu}$ has flat boundary conditions. In the $\omega \to \infty$ limit GR is reobtained, giving an accurate description of the universe. The spatial variation of $G$ is given by local distribution of matter and to infinity $\Phi \to \Phi_0$. On cosmological scales differences between GR and Brans-Dicke theory do arise. These differences leaves experimentalists with a task to determine which theory describes the universe best.

1.4 Experimental Bounds

In the last section we discussed the limit of $\omega \to \infty$ to compare the theory with GR. In the early universe solutions with a smaller coupling constant are clearly more interesting, since large deviation from GR gives rise to new and interesting physics. But if we take for example $\omega \approx 1$, as initially proposed by Brans and Dicke [1], the gravitational constant would be changing more rapidly than observed today. We will shortly discuss some experiments in which predictions of Brans-Dicke theory contradict those from GR. As
we will see, this gives important constraints on the coupling constant. To quantify the deviation from an alternative theory of gravity to GR, Will and Nordtvedt introduced the Parametrized Post-Newtonian (PPN) formalism [7]. In this formalism a number of parameters are introduced, from which $\gamma$ and $\beta$ are the ones important for our discussion. The first describes to what extent spatial curvature is produced by a unit mass and is zero in the case of Newtonian physics and one in the case of GR. If gravity is governed by other laws of nature like Brans-Dicke theory, the scalar field changes the effect of the metric and $\gamma$ is no longer exactly one but smaller. The second parameter is a measure for the non-linearity of the theory in the superposition law for gravity, also one in case of GR. In Brans-Dicke theory the parameters are given by [7],

$$\gamma = \frac{1 + \omega}{2 + \omega}, \quad \beta = 1.$$  \hspace{1cm} (1.26)

In case of the more general scalar-tensor theory, the appearance of a varying coupling $\omega(\Phi)$ results in a different value for $\beta$,

$$\beta = 1 + \Lambda \beta = 1 + \frac{d\omega/d\Phi}{(3 + 2\omega)^2(4 + 2\omega)}(\Phi_0),$$  \hspace{1cm} (1.27)

with $\Phi_0$ the value of the scalar today. Since $\beta \neq 1$ is not a feature of pure Brans-Dicke theory, we will not go very deep into the possible detection methods. If the inequality holds, it should be seen in the perihelion shift of Mercury or in the Nordtvedt effect [8]. This states that the gravitational self-energy of a certain body contributes to its gravitational mass but not to its inertial mass. This would cause a difference in gravitational acceleration of the earth and moon in the sun’s gravitational field, for instance. It also implies violation of the strong equivalence principle.

1.4.1 Several experiments

One of the most successful models in cosmology is nucleosynthesis which explains the primordial abundances of elements in the universe. In this model, the $^4$He abundance is mostly determined by the number of neutrons present at the moment of deuterium formation. Deuterium formations happens at the time $\beta$ decay becomes less important and is dependent on the temperature $T_W$ of the weak interaction freeze-out. This freeze-out happens when the weak interaction rate becomes smaller than the Hubble rate $H$, dependent on $G$. In other words, $H$ and thus $G$ provides the timescale on which the processes important for nucleosynthesis act. We can therefore conclude that the generalization of BD theory to varying $G$ must have its influence on nucleosynthesis. It is found by Clifton et al that the constraints from nucleosynthesis demand $\omega > 332$ without considering a cosmological constant, $\omega > 277$ when including a cosmological constant [9]. Lower values of
1.4. EXPERIMENTAL BOUNDS

ω would make G vary too much. Nucleosynthesis can become too efficient for instance when G is just slightly larger. As a result, heavier elements would be overproduced, which is clearly not observed.

Already in 1969, lunar missions in the Apollo program placed a field of reflectors on the moon. It turned out to be possible to detect reflected laser pulses on a ground based observatory. This gave the possibility of distance measurements with an incredible accuracy (±3 cm), sufficient for a whole class of tests on General Relativity. Kepler’s third law states that the variation of the mean motion $\bar{v}$ and semimajor axis $a$ of an orbiting object are connected through,

$$2\frac{\dot{\bar{v}}}{\bar{v}} + 3\frac{\dot{a}}{a} = \frac{\dot{G}}{G}.$$  \hspace{1cm} (1.28)

Variation of $G$ implies therefore variation of the mean distance and period. One has to take into account that energy dissipation due to tidal effects also increases the earth-moon distance and lunar period. This effect is expected to be at least 1-2 orders of magnitude larger than variations due to $\dot{G}/G$. It is therefore necessary to incorporate a very detailed model of tidal effects. This has been done most recently by Williams et al, resulting in a rate of change [10],

$$\left| \frac{\dot{\Phi}}{\Phi} \right| = \left| \frac{\dot{G}}{G} \right| = (4 \pm 9) \times 10^{-13} \text{yr}^{-1}.$$  \hspace{1cm} (1.29)

The variation $\dot{G}/G$ of the Gravitational 'constant' is roughly identified with the PPN parameters in the following way,

$$\frac{\dot{G}}{HG} \simeq 4\beta - \gamma - 3 = \frac{1}{2 + \omega} \quad \text{(BD)}.$$  \hspace{1cm} (1.30)

This would suggest $\omega > 20,000$.

Another interesting situation arises in binary systems. Although this method is not yet to compete with other experiments, it is theoretical possible to obtain a very high accuracy in estimating $\omega$. It relies on the property that GR and BD theory both predict different values for the rate of change of the periods due to energy dissipation by gravitational waves. This is mainly due to the fact that, as pointed out in section 1.2.2, also scalar gravitational waves occur in BD theory. Inertial masses depend on the value of $G$ and thus a massive body is rather described by a mass function of the form $M(\Phi)$. The resulting discrepancy between inertial and gravitational mass leads to the Nordtvedt effect as pointed out before. A quantity called sensitivity $s$ describes the strength of this effect can be assigned to any massive body.
and is defined,
\[ s = 2 \frac{d \log(M)}{d \log(\Phi)}. \] (1.31)

It is a measure of the gravitational binding energy per unit mass of the body under consideration; that is, about $10^{-6}$ in case of an ordinary star and $1/2$ in the limiting case of a black hole. When two bodies of sensitivity $s_1$ and $s_2$ orbit around each other, not only the usual metric quadrupole gravitational waves are emitted, but scalar dipole radiation as well. When $s_1 \neq s_2$, the centre of gravitational mass does not coincide with the centre of inertial mass (Eardley 1975). As a result, the centre of inertial mass, given by the sensitivity, will perform a rotation around the centre of gravitational mass. The varying dipole moment is a source of scalar gravitational waves, and the total loss of energy of such a system is then given by [11],
\[ \frac{dE}{dt} = -\frac{\mu^2 m^2}{r^4} \left[ \frac{8}{15} (\kappa_1 v^2 - \kappa_2 \dot{r}^2) + \frac{1}{3} \xi (s_1 - s_2)^2 \right]. \] (1.32)

Here $m$ and $\mu$ denote the physical and reduced mass, respectively, $r$ is the orbital radius and $v$ the velocity, $\kappa_{1,2}$ and $\xi$ are constants. In GR $\kappa_{1,2}$ are 12 and 11 respectively and $\xi = 0$, but in BD theory they all receive corrections proportional to $(2 + \omega)^{-1}$. The last term is the energy dissipation due to scalar radiation of gravitational energy and the first two due to the familiar metric quadrupole gravitational radiation. The energy dissipated through scalar radiation may for certain parameters even be larger than through quadrupole waves. The observable result is a more rapid decay of the orbit, with increasing effect when $s_1 - s_2$ is larger. It is theoretically possible to put very high constrains on BD theory when observing a binary system with the right properties [12]. One of the benefits of this experiment is that it can yield a test in the strong field regime, while all other tests are within the weak field regime.

The bounds most commonly referred to are coming from measurements that make use of the fact that photons get bent and delayed in the curvature produced by a large mass. This effect is proportional to $\gamma + 1$, with the Post-Newtonian parameter $\gamma$ as given in eqn. (1.26). The most recent experiment was performed with the Cassini spacecraft in 2002 [13]. At a distance of 8.43 AU the spacecraft passed (almost) behind the sun in the line of sight from the earth while transmitting radio signals to a ground based observatory. The frequency shifts were measured and indicated a value of $\gamma = 1 + (2.1 \pm 2.3) \times 10^{-5}$ [13]. That means the lower bound to $\omega$ is set to $\omega > 40,000$. Till now this is the most stringent constraint of the theory. General scalar-tensor theories and scalar theories of gravity in string theory involving the dilaton predict that the deviation from GR should be about
1.5. COSMOLOGICAL FEATURES IN BRANS-DICKE THEORY

\[10^{-5} < \gamma < 10^{-7}\] [14], which is nearly reached by this experiment.

Many experiments do suggest a very large value for \(\omega\). They are based on measurement of the variability of the gravitational 'constant' as observed today, since a large \(\omega\) implies slow dynamics of the field. In this scenario the scalar will approximately equal the current gravitational constant just after inflation, from when it will hardly evolve any further. Several cosmological applications of Brans-Dicke theory demand \(\omega\) to be much smaller, though. Besides considering the more general scalar-tensor theories, a possible solution is extending the original theory with a potential. This can anchor the value of \(\Phi\) to the Planck mass squared, giving the late time solution of \(G_0/\Phi_0 = G\). In such a case, the value of \(\omega\) can be anything, not bounded by the current observations. Another benefit of this scenario is that the dynamics of \(\Phi\) at early times can be less trivial as well.

1.5 Cosmological features in Brans-Dicke theory

1.5.1 Extended Inflation

The inflationary model was once invented to solve a collection of serious problems that cosmology had. First of all, there was a lack of explanation why the universe appears to be so smooth on large scales (horizon problem) and has such a rich structure on small scales (small-scale inhomogeneities). Furthermore, from all possible curvatures, the universe has 'chosen' to be very nearly flat at present day, although its curvature should have been enormous at the Planck time (flatness problem). Finally, modern particle physics suggests that at early times and high temperature a huge amount of stable, heavy particles like gravitinos should have been produced, closing the universe several times. Since this is not the case, we somehow have to get rid of these unwanted relics. Inflation, first proposed by Alan Guth [15], can solve all these problems. The basic idea is that in the earliest stage of the universe a period of domination of vacuum energy occurred. During this de Sitter phase the scale factor grows exponentially while curvature remains constant. A causal sphere, that is, smaller than a Hubble volume, can inflate to a size much larger than our present visible universe. This explains the horizon problem and flatness problem. The relics are diluted by the enormous stretching of space. In the original models of inflation the inflaton, the particle responsible for the inflation, obtains a large vacuum energy. This can be achieved in two ways. The inflaton could be trapped in a local minimum at zero expectation value, the false vacuum state. After tunneling it rolls down to the true ground state at nonzero expectation value. This process is a first order phase transition and occurs through nucleation of bubbles in which the true vacuum state is reached. The problem arising in this scenario is that space between these bubbles is still trapped in the local
minimum and hence expanding inflationary. The rate at which bubbles of true vacuum are nucleated by tunneling is not high enough to overcome this expansion and inflation will last forever. This is known as the graceful exit problem. Another possibility is a very flat potential for the inflaton. Since timescales in the early universe were very short, the timescale at which phase transitions occur are nontrivial. The inflaton could start from a metastable maximum and slowly roll off the flat potential towards a true ground state. This is a second order phase transition. Getting the exact amount of inflation, the potential has to be fine tuned to have the right properties, however.

In extended inflation [16] a first order phase transition is still possible without the bubbles nucleating at a too slow rate. In Brans-Dicke theory the gravitational constant time dependent. As a result the Friedman equation of motion of the scale factor changes to,

\[ H^2 = \frac{8\pi \rho}{3\Phi} - \frac{k}{a^2} + \frac{\omega}{6} \left( \frac{\dot{\Phi}}{\Phi} \right)^2 - H \left( \frac{\dot{\Phi}}{\Phi} \right) . \]  

Here we used the Brans-Dicke scalar \( \Phi \) defined as in the action in eqn. (1.4), with equation of motion from eqn. (1.9),

\[ \ddot{\Phi} + 3H \dot{\Phi} = \frac{8\pi(\rho - 3p)}{3 + 2\omega} . \]  

As can be seen, the trace of the stress energy tensor is taken to be that of an ideal fluid. We take the energy density dominated by the false vacuum energy density and assume a flat universe in which \( k = 0 \). To solve the flatness problem a nonzero value of \( k \) is preferable, but these solutions quickly converge to the \( k = 0 \) solution. The equation of state during inflation is \( p \approx -\rho \) and the equations of motion are solved by,

\[ \Phi = M_P^2 \left( 1 + \frac{\chi t}{\alpha} \right)^2 \]  

\[ a(t) = \left( 1 + \frac{\chi t}{\alpha} \right)^{\frac{\omega+1}{2}} . \]  

Here is \( M_P \) the Planck mass, \( \chi^2 = 8\pi \rho_f / 3M_P^2 \) is the Hubble constant squared in the Einstein theory of gravity and \( \alpha^2 = (3 + 2\omega)(5 + 6\omega)/12 \), which approaches \( \omega^2 \) in the limit of large \( \omega \). For small times \( \chi t < \alpha \) and large \( \omega \) the scale factor \( a(t) \approx e^{\chi t} \) behaves exponentially and the Ricci scalar is constant in time. This solution corresponds to a de Sitter universe as we are familiar with. For later times the scale factor becomes powerlaw. The rate at which bubbles of true vacuum nucleate changes however. As a result, the conversion of space to the ground state still occurs exponentially in time, while the expansion is only powerlaw. The inflation cannot keep up and soon the whole universe is dominated by true vacuum. Inflation has ended. The false vacuum energy is converted into bubble wall energy and
dissipates to a thermal distribution of particles by collisions. Oscillation of
the inflaton around the true ground state reheats the universe and evolution
proceeds in the same way as in conventional inflation models. A problem
arising in this scenario is the nucleation of too large bubbles [17, 18]. The
dimensionless parameter describing the efficiency of the bubble nucleation
is given by,

$$\epsilon = \frac{\Gamma}{H^4}. \quad (1.36)$$

The slow variation of $H$ will eventually push $\epsilon$ to a value of order one, which
is the criteria to end inflation. Since $\Gamma$ changes slowly in time as well, though,
there will still be some bubbles nucleating at times as early as 50 e-foldings
before the end of inflation. These bubbles have the possibility of growing
very large and if this was the case then there would be a prominently im-
printed signature of them in the CMBR, which is not observed. To avoid
this so called big bubble problem, $\omega$ cannot be larger than 20. This value is
not supported by experimental bounds but the value of $\omega$ might have been
smaller during inflation as discussed before.

An interesting variation to extended inflation contains a curvature coupling
of the inflaton [19]. A term like this is natural since no new fields are intro-
duced and terms like this can even be generated by quantum gravitational
corrections when absent in the bare action [20]. Consider an action of the
following form,

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} \zeta \phi^2 R + \frac{1}{2} \partial^\mu \phi \partial_\mu \phi + \frac{1}{2} \partial^\mu \sigma \partial_\mu \sigma + V(\sigma) + \frac{1}{2} \xi \sigma^2 R \right]. \quad (1.37)$$

Here $\sigma$ denotes the inflaton with curvature coupling constant $\xi$ and potential
$V(\sigma)$. The Brans-Dicke scalar from eqn (1.4) is redefined as \( \phi^2 = 8\omega \Phi \). This
redefinition is often made in the literature since $\Phi$ is a dimension two field
and the kinetic term gets a canonical form in terms of $\phi$. Adding a potential
for $\phi$ would make the fields appear almost symmetrical, although since $\zeta$ is
negative, the curvature couplings appear with different signs. The effective
potential of the inflaton now appears with a metastable minimum due to
the curvature coupling. In Brans-Dicke theory the Ricci scalar turns out to
be a function of the inverse of the Brans-Dicke scalar during the trapped
phase. Eventually $R$ will drop down to a certain critical value for which
the local minimum disappears. At that moment the inflaton will roll down
the potential to the true vacuum state much in the same way as described
above. This corresponds to a second order phase transition. The amount of
e-foldings the universe undergoes during the trapped phase and rolling phase
depends on the various coefficients of the theory. As further examined in
ref. [19], two possibilities are of special interest: the number $N_r$ of e-foldings
during the rolling phase smaller and larger than 60. In the first case, scales
corresponding to the present observable universe crossed the Hubble radius
during the trapped phase. This means large scale structure formation was
CHAPTER 1. BRANS-DICKE THEORY.

seeded by perturbations during this first phase. In the second case no present observable scales were generated during the trapped phase and this phase only served for giving the right initial conditions to the rolling phase. The latter one produced all the density perturbations leading to the structure in the universe as we see it.

1.5.2 Density perturbations

As pointed out in the previous section, inflation gives an explanation for the large scale homogeneities. With density perturbations we mean the small scale inhomogeneities at very early times that seeded the formation of large scale structures like galaxies today. From WMAP it has become clear that the density perturbations, that left their imprint in the CMBR, are nearly scale invariant with \( \frac{d\rho}{\rho} \approx 10^{-5} \). Zel’dovich [21] and Harrison [22] already argued that a (nearly) scale invariant spectrum of density fluctuations was needed to seed the formation of structures like clusters, galaxies and stars. The mechanism to produce such a spectrum during inflation was explored by Chibisov and Mukhanov and Hawking and Starobinsky. The scalar field responsible for the exponential expansion, the inflaton, is subject to quantum fluctuations that can be decomposed in Fourier modes with physical wave number \( \frac{k}{a(t)} \). Here denotes \( a(t) \) the scale factor. As the universe expands, the physical wave number decreases so the length scale associated with it will increase and eventually become larger than the Hubble radius \( H^{-1} \). This is called the first Hubble crossing or freeze out and from this point on quantum fluctuations cannot modify anymore the amplitude. The mode evolves classically while superhubble and eventually re-enters the Hubble radius when inflation has ended and the Hubble radius starts to increase. The \( k \)-dependence of the amplitude of the initial quantum perturbations turns out to be weak. Much more important is the time at which a certain mode freezes out, since this determines the second Hubble crossing and thus the final scale of the mode. Further evolution is scale independent and hence all scales are present with nearly equal amplitude, the Harrison-Zel’dovich spectrum. As an example, scales corresponding to the current observable universe became superhubble at about 57 e-foldings before the end of inflation. This sets the minimal amount of inflation needed to solve for the large scale homogeneity. Perturbations corresponding to the scale of galaxies can become superhubble at about 48 e-foldings before the end of inflation. These figures can differ in several models, however, since it also depends on the post-inflationary evolution.

To investigate density perturbations in Brans-Dicke theory, we will have to consider extended inflation as the inflationary mechanism. Density perturbations may then arise from fluctuations of the BD scalar as well as from bubble collisions after the phase transition that ended inflation (see previ-
1.5. COSMOLOGICAL FEATURES IN BRANS-DICKE THEORY

ous section). The latter will not be considered here, since models in which bubbles do not affect the CMBR anisotropy are favorable. To investigate the first, it is convenient to transform to the Einstein frame since extended inflation then resembles slow roll inflation with the BD scalar as inflaton and the Higgs field as order parameter of the phase transition. The most important change is the different definition of the scale factor and the resulting time dependence of $H$. As a result the fluctuation amplitudes of $\Phi$ are not exactly scale independent but rather $\delta \Phi/\Phi \propto \lambda^{4/(2\omega-1)}$ [23], with $\lambda$ the length scale of the fluctuation. This results in a power spectrum of the form,

$$P(k) \propto k^{1-8/(2\omega-1)} = k^{n_s}.$$  \hspace{1cm} (1.38)

A scale independent spectrum is obtained in the limit $\omega \rightarrow \infty$. From Spergel et al [24] the most recent value of the slope of the power spectrum obtained from WMAP observations yields $n_s = 0.95 \pm 0.02$. That implies a value of $\omega \approx 80$.

1.5.3 Dark energy

The universe seems currently to be in an accelerating phase. A possible explanation is hypothetical dark energy which is currently the dominating energy contribution [25], [26]. Recent observations from type Ia supernovae indicate that 70 percent of the energy density of the universe is dark energy. Already a long time ago the Einsteinian cosmological constant revived and was added to the theory of general relativity because there was no reason for setting it to zero. Such a source of dark energy or vacuum energy has a negative equation of state, giving the universe a negative pressure and forcing it to an accelerating expansion. There was however no obvious reason why the value of the energy density was so small, the so called Cosmological Constant Problem. As a solution it was proposed by Dolgov (1983) that dark energy might vary in space and time and can therefore be described by a scalar field. Since then many theories of a dynamical cosmological constant developed under the name Quintessence (Steinhardt 1998), or K-essence in case of scalar fields with non-canonical kinetic terms.

Several attempts have been made to incorporate Brans-Dicke theory into a model of dark energy and matter. The original BD action (1.4) contains already a non-canonical kinetic term and resembles therefore a theory of k-essence. In the previous section it was shown that the BD scalar slows the exponential expansion during inflation down to powerlaw. The theory can in a similar way have its effect in other expansion phases of the universe. In one study it was realized that a universe with dominating BD scalar exhibits no acceleration nor deceleration. Furthermore, the theory can decrease the deceleration and acceleration in their respective eras and hence functions as
a catalyst to accomplish a smooth transition between the two [27], [28]. It was also argued that the scalar field in combination with matter acts like dark matter and in combination with a cosmological constant as dark energy. In this way a unified model of dark matter and energy is obtained with the features of both catalyzed by one single field. Models like these have an effect on the age of the universe however, since the intermediate phase of BD scalar domination extends the lifetime
Chapter 2

Baryogenesis.

2.1 General Outline.

When looking around it is clear the universe we live in contains vast amounts of matter. It is one of the most challenging subjects in Cosmology to explain where this matter is coming from and, moreover, to explain the absence of anti-matter in the mean time. Experimental data confirms a baryon asymmetry in favor of baryons over anti-baryons. In our solar system anti-protons are found at a ratio of $10^{-4}$ in cosmic rays, but those are all secondary, products due to collisions of cosmic rays with the interstellar medium. The strongest constraint on the asymmetry comes from Big-Bang nucleosynthesis and WMAP, which set the baryon number density $n_B$ to entropy density $s$ ratio to \[ n_B/s \equiv (6.1 \pm 0.3) \times 10^{-10}. \] (2.1)

to a certain extent, the laws of physics seem to treat particles similar to anti-particles, so one might suspect anti-matter to lurk somewhere in the universe in the same amount as matter. However, annihilation processes on boundaries between domains containing matter and anti-matter would produce enormous bursts of gamma radiation, which is never observed. Even a situation with large separation between such areas would be impossible. Baryons and anti-baryons were still in thermal equilibrium till the temperature of the universe dropped to $T \sim 22 MeV$. At that temperature, due to annihilation processes the baryon to entropy ratio already dropped to the small value of $n_b/s = n_{\bar{b}}/s \sim 10^{-19}$ - about 9 orders of magnitude smaller than the observed ratio today. One could propose a yet undiscovered mechanism to separate matter and anti-matter at higher temperatures when $n_b/s$ was larger. At temperatures of about $T \sim 38 MeV$ the (anti)-particles remain in chemical equilibrium at the desired value of eqn. (2.1). But at that age every sphere of the Hubble radius in the universe contained only about $10^{-7} M_\odot$, making separation of amounts of matter and anti-matter at the order of galaxies causally impossible. It seems almost inevitable that nature
CHAPTER 2. BARYOGENESIS.

did indeed favor matter over anti-matter and thus created the observable universe with its entropy ratio.

2.2 Shakarov Conditions.

It is very unsatisfying to accept eqn. (2.1) as an initial condition of the universe. If an asymmetry like this occurs, a dynamical process in the very early universe should be the origin of it. In 1967 Shakarov proposed three conditions needed to generate a baryon asymmetry:

- Baryon number violating interactions have to occur. If one would like to have any baryon at all, one should better have a process in which net baryon number is produced. Baryons and anti-baryons both carry positive baryon number however, and without other violating processes an equal amount of both is produced. We will need more;

- C and CP violating processes have to occur. The violation of charge conjugation (C) together with combined violation of parity and charge conjugation (CP) would have the possibility to push the baryon production towards an asymmetry. But in thermal equilibrium the densities are completely defined by the energy of the particles and since that is the same for both baryons and anti-baryon we have \( n_b = n_{\bar{b}} \) and no asymmetry survives. Hence the last condition;

- Departure from thermal equilibrium. This will guarantee the densities \( n_b \) and \( n_{\bar{b}} \) to be unequal. Without production of entropy at later times, the entropy ratio \( n_b/s \) will remain constant.

We can realize these conditions in the following toy model [30]. Consider a bath with equal amounts of particles \( X \) and its antiparticles \( \bar{X} \). Both can decay to a combination of quarks and leptons, as indicated in table 2.2, with different branching ratios. Note that these processes are baryon number violating, since the two final states differ in baryon number. CPT invariance requires the total decay ratios of \( X \) and \( \bar{X} \) to be equal, but C and CP can be violated and if this is the case, \( r \neq \bar{r} \). The net baryon number produced by these processes is given by,

\[
\epsilon \equiv B_X + \bar{B}_X = \left[ \frac{2}{3} r + \frac{1}{3} (1 - r) \right] + \left[ \frac{-2}{3} \bar{r} + \frac{1}{3} (1 - \bar{r}) \right] = r - \bar{r}.
\]

(2.2)

Our initial bath is of a very high temperature such that the abundances are in thermal equilibrium. In this case the number density of the (anti)particles resembles the number density of photons. The decay processes as described can occur freely back and forth as long as the temperature remains above the threshold given by the mass of the \( X \). When the universe evolves,
temperature will eventually drop below this threshold and the $X$ and $\bar{X}$ become overabundant. As the temperature decreases further and further, $T \ll m_X$ and the bosons freely decay in the baryon number generating way described above. The back reactions are suppressed by the low temperature. This whole system is called the out-of-equilibrium scenario and describes how the third Sakharov condition is often fulfilled. As long as $C$ and $CP$ is violated and thus $\epsilon \neq 0$ this scenario can produce net baryon number. There is a catch concerning $CP$ violation however, as will be explained in the next section. All kinds of models for baryogenesis in different sectors of particle physics have been proposed in the past. We will discuss in short the most important ones.

$$\begin{array}{cccc}
\text{particle} & \text{final state} & \text{branching ratio} & B \\
X \to & q\bar{q} & r & \\
X \to & \bar{q}\ell & 1 - r & - \\
\bar{X} \to & \bar{q}\bar{q} & \bar{r} & - \\
\bar{X} \to & q\ell & 1 - \bar{r} & - \\
\end{array}$$

Table 2.1: Final states and branching ratios for $X$ and $\bar{X}$ decay.

2.3 GUT Baryogenesis

The first proposed models of baryogenesis were based on Grand Unification Theories, GUT’s. Although still under consideration, GUT’s can provide a basis in which baryogenesis is achieved. In GUT’s the electroweak and strong interactions are unified in one single gauge theory represented in a simple or semi-simple non-Abelian symmetry group like SU(5) or SO(10). Although it turned out, as we will see, that the SU(5) theory actually fails to describe a correct mechanism, it is the easiest theory to illustrate how GUT baryogenesis works in principle.

2.3.1 Short introduction to SU(5) GUT

This model was first proposed by Georgi and Glashow [31] to unify the three groups of the Standard Model to one symmetry group at sufficiently high temperatures. The motivation came from the observation that the coupling strengths of the three fundamental forces except gravity tend approximately towards an equal strength when approaching the high energy scale $M_{GUT}$. This fact makes it tempting to propose that beyond $M_{GUT}$ all forces are united to one. The representations $\bar{10}$ and $10$ associated with SU(5) can be identified with the (lefthanded) quark doublet $Q$, the singlets $u^c$ and $d^c$, the
lepton doublet $L$ and the singlet $e^c$ in the following way,

$$
\bar{5} \rightarrow (3, 1)^{\frac{1}{3}} \otimes (1, 2)^{-\frac{1}{2}} \quad (d^c, L)
$$
$$
10 \rightarrow (3, 2)^{\frac{1}{5}} \otimes (3, 1)^{-\frac{3}{2}} \otimes (1, 1)^{1} \quad (Q, u^c, e^c).
$$

(2.3)

Here the $(Q_c, Q_L)Q_{Y/2}$ denote the color charge, weak isospin and hypercharge of the $SU(3)_c \times SU(2)_L \times U(1)_Y$ Standard Model gauge group respectively. For one generation the two representations have the following form,

$$
\bar{5} = \begin{pmatrix}
 d^c_1 \\
 d^c_2 \\
 d^c_3 \\
 e \\
 \nu
\end{pmatrix},
\quad
10 = \begin{pmatrix}
 0 & u^c_2 & -u^c_1 & Q^1_1 & Q^2_1 \\
 -u^c_2 & 0 & u^c_3 & Q^1_2 & Q^2_2 \\
 u^c_1 & -u^c_3 & 0 & Q^3_1 & Q^3_2 \\
 -Q^1_1 & -Q^2_2 & -Q^3_3 & 0 & e^c \\
 -Q^1_2 & -Q^2_3 & -Q^3_4 & -e^c & 0
\end{pmatrix}.
$$

(2.4)

Here the lower indices are color indices and $Q^1_i = u_i$, $Q^2_i = d_i$. Super symmetry is not considered in this example, since it is not needed to point out the properties of GUT’s of interest for baryogenesis. Similar to the electroweak transition, a Higgs like scalar field $\Phi$ is added to break the symmetry down to the Standard Model gauge group. The expectation value of $\Phi$ is given by $\langle \Phi \rangle = \text{Diag}(2v, 2v, 2v, -3v, -3v)$. The generators that commute with $\langle \Phi \rangle$ are unbroken and correspond to the generators of the SM gauge group. Those that do not commute are broken and the associated vector bosons obtain a mass. Some of them mediate processes between leptons and quarks and necessarily violate baryon number. An example of such a generator representing the baryon number violating boson $X$ is,

$$
X = \begin{pmatrix}
 0 & 0 & 0 & 1 & 0 \\
 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 \\
 1 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0
\end{pmatrix}.
$$

(2.5)

From (2.4) it is easily seen $X$ couples to $d^c$ and $e$ and there is a coupling to $Q$ and $e^c$ as well. A far reaching result is the possible proton decay. Since proton decay is not something we observe daily (fortunately), the responsible mediating gauge boson $X$ has to be extremely heavy meaning that the expectation value $v$ is very large. Current experimental data suggest a lifetime of $\tau \gtrsim 10^{34}$ yr, which implies a mass $m_X \gtrsim 10^{16}$ GeV. This large mass makes the process nowadays highly unlikely and hence the large value of $\tau$, but once the temperature of the universe exceeded $m_X$ and baryon number violating processes like these were common. The value of $\tau$ does not match the predictions of GUT based on $SU(5)$, being some orders of magnitude smaller, which is one of the reasons why this theory is ruled out. SO(10) is still possible, although on the edge of exclusion [32], and possesses
similar properties as described in this section. The benefit of SO(10) over SU(5) is also the fact that the latter does not violate $B - L$ while the first does. B-L violation is more preferable than just B violation as will become clear when discussing Electroweak baryogenesis. SO(10) makes also the coupling strengths to converge better to an equal strength (within 2 sigma) when going to higher energies.

The vector boson $X$ is also CP violating and together with the expansion of the universe that accounts for the departure of thermal equilibrium the three Sakharov conditions are satisfied. There is still a catch though. On tree level, the CP violating phase cancels out between the distinct end states of the boson decay. This means in the out of equilibrium decay mechanism of the previous section $r$ equals $\bar{r}$ in table 2.2 and no net baryon number is produced. When going to higher order contributions, the exchanged particle can be on the mass shell, resulting in a non vanishing phase. It is only in this loop corrections that CP violation becomes effective and baryon number is generated. It has been shown this is not necessarily the case when considering coherent baryogenesis [33], though.

2.3.2 Preheating

Any baryons produced during inflation are expected to dilute completely due to the exponential expansion. Therefore GUT baryogenesis can only occur after inflation, but in most scenarios for reheating the universe cannot have a temperature greater than $10^9\text{GeV}$. The reason is that a GUT is build on supersymmetry that predicts a whole new family of super symmetric particles like the gravitino. If after inflation the inflaton decays and reheats the universe to temperatures well above $10^9\text{GeV}$, too many of these long-lived particles would be created and close the universe in a way we do not observe today. Hence the driving force of the model will keep the universe from reheating to the threshold where baryogenesis can re-ignite again. A possible solution to this problem is given by the preheating scenario [34]. In this case the $\chi$ boson is coupled to the inflaton $\sigma$ in a bi-quadratic coupling such that the potential $V(\chi, \sigma)$ yields,

$$V(\chi, \sigma) = -\frac{1}{2}\sigma^2 + \frac{\lambda}{4}\sigma^4 - \frac{g^2}{2}\sigma^2\chi^2.$$  \hspace{1cm} (2.6)

As a result, the equation of motion for $\chi$ is of the form of a Matthieu function and obtains an oscillatory effective mass during reheating when the inflaton oscillates around the minimum of its potential. Coherent decay of the inflaton to $\chi$ particles is then possible even when $m_\chi$ is 100 times larger then the inflaton mass ($\approx 10^{13}\text{GeV}$) and kinematic decay is impossible. This mechanism is very effective and within a short oscillating era the inflaton can decay and fill the universe with a large amount of $\chi$ bosons. As the temperature drops, the out of equilibrium decay can provide the baryon
CHAPTER 2. BARYOGENESIS.

generation as pointed out above. Similar models are constructed with negative bi-quadratic coupling of the inflaton and another scalar field [35] and with a Yukawa coupling to fermions [36]. In the latter case, an even more efficient model can be constructed, generating masses up to $10^{18}$ GeV [37]. This model can also be interesting in case of Leptogenesis, to be considered in section 2.6, in which an overabundance of very heavy right handed neutrinos is needed.

2.4 Electroweak Baryogenesis

2.4.1 Sphaleron processes

The three Sakharov conditions for baryogenesis are found in the Standard Model as well. At first sight, the symmetries of the Standard Model seem to forbid B and L violation at both tree level and any order of perturbative expansion. B and L violating processes only occur in dimension six and five operators respectively, both suppressed by a very high energy scale. 't Hooft [38] found out that instantons in non perturbative processes do violate B+L however, while conserving the orthogonal B-L. This effect is very tiny and at present day temperature these processes are highly suppressed, making the proton very stable. The underlying framework is based on the anomaly occurring at quantum level in the baryon and lepton current,

$$\partial_\mu J_B^\mu = \partial_\mu J_L^\mu = n_f \left( \frac{g^2}{32\pi^2} W^a_\mu \tilde{W}^a_\mu - \frac{g'^2}{32\pi^2} F^\mu_\nu \tilde{F}^\mu_\nu \right).$$  \hspace{1cm} (2.7)

Here $n_f$ are the number of families, $g$ and $g'$ are the couplings of SU(2)$_L$ and U(1)$_Y$ respectively and $J_B^\mu$ denotes the baryon number current. Furthermore, $F^\mu_\nu$ is the U(1)$_Y$ field strength tensor, $W^a_\mu$ are the SU(2)$_L$ field strength tensors and the tilde denotes the dual field. As can be seen, these currents do not obey perfect conservation conditions but are anomalous. In abelian theories this leads to a correction of the B (L) current to make it conserved, but due to its rapid decay, integrating over all space makes it vanish. In non-abelian gauge theories, non perturbative field configurations add extra terms to the correction, leading to violation of B and L. These configurations are called instantons. To clarify further what is happening, consider the vacuum states of the theory. In an abelian theory, fixing the gauge leaves us with only one unique ground state, the trivial solution in which the vector potential vanishes. In non abelian gauge theories, a class of gauge transformations occurs, labeled by integers, in which the energy vanishes. These have to be considered as periodic, distinct vacuum states, separated by an energy barrier and in our case at hand labeled by the Chern-Simons number $n_{CS}$,

$$n_{CS} = \frac{g^2}{32\pi^2} \int d^3 x \epsilon^{ijk} Tr \left( A_i \partial_j A_k + \frac{2}{3} i g A_i A_j A_k \right).$$  \hspace{1cm} (2.8)
This number can be interpreted as a winding number. This vacuum configuration resembles the vacuum structure of a Yang-Mills theory. A transition from a certain vacuum state to the next changes the Chern-Simons number by one and because of the anomaly the baryon and lepton numbers change as $\Delta \bar{B} = \Delta L = n_f$. This means a transition to a neighboring vacuum with higher CS number creates nine (left handed) quarks and three leptons, one for each generation and color. The height of the energy barriers are determined by the field configuration which correspond to sitting on top of the barrier. This solution is called the sphaleron, having one negative eigenvalue corresponding to rolling down the potential into one of the two neighboring wells. The sphaleron energy is given by,

$$E_{\text{sph}} = \frac{c}{g^2} M_W.$$  \hfill (2.9)

The constant $c$ is to be determined by detailed calculations, $g$ denotes the weak coupling and $M_W$ the electroweak mass scale. In the low temperature limit, transitions can only take place by tunneling. As stated before, these processes are exponentially suppressed and so is the baryon and lepton number violation. At high temperatures like during the early universe however, the crossing rate is larger due to thermal fluctuations and yields,

$$\Gamma_{\text{sph}} \approx T^4 e^{-E_{\text{sph}}/T}.$$  \hfill (2.10)

This result is only valid after the electroweak phase transition when the W boson acquired a mass. Above about 100 GeV the electroweak symmetry is restored and calculating the transition rate extremely more difficult. Current lattice calculations yield, [39] [40],

$$\Gamma_{\text{sph}} \approx 20 \alpha_W^5 \log(\alpha_W^{-1}) T^4.$$  \hfill (2.11)

Sphalerons are important in any theory of baryogenesis. When producing a current carrying B, one has to take into account sphaleron processes, since these can wash out the condensate carrying baryon number. Generating an orthogonal current B-L will be fine, since sphalerons are B-L conserving. This process is used in leptogenesis, in which a condensate carrying only L is processed to a B asymmetry.

### 2.4.2 CP violation in extended models

Concerning the second Sakharov condition, C is already maximally broken in SU(2)$_L$, since only lefthanded leptons couple in it. Also CP is known not to be an exact symmetry of the theory. The charged current in the electro weak current contains a mass mixing matrix known as the Kobayashi-Maskawa mixing matrix. It contains one independent complex phase when considering three generations of particle species. Unfortunately, the corresponding
CHAPTER 2. BARYOGENESIS.

effects are multiplying this phase by a dimensionless constant of order $10^{-20}$. This turns out to be much too small to create a baryon asymmetry as large as observed in the universe. Supersymmetry can solve this problem. In the minimal supersymmetric standard model (MSSM) new sources of CP violation appear. These are: 1) Trilinear couplings of the two Higgs doublets (supersymmetry needs two Higgs fields in order to be able to give all quarks and leptons mass and to cancel anomalies, see appendix A.2), quark and lepton doublet and quark and lepton singlet; 2) Bilinear coupling of the Higgs doublets; 3) Gaugino mass matrix; 4) Soft scalar mass matrix. By redefinitions of fields many phases can be removed but two. The listed sources of CP violation are all on tree level. In the MSSM, CP violation is also found in the interactions of the Higgs fields with the superfields charginos, neutralinos and stops (super partners of charged and neutral gauge bosons and the top quark) at one loop level when supersymmetry breaking occurs. The terms of interest are,

\[ V_{cp} = \lambda_1 (H_1 H_2)^2 + \lambda_2 |H_1|^2 H_1 H_2 + \lambda_3 |H_2|^2 H_1 H_2 + \text{h.c.} \quad (2.12) \]

The Higgs fields can be written in the unitary gauge to be,

\[ H_1 = (\varphi_1, 0) \quad \text{and} \quad H_2 = (0, \varphi_2 e^{i\theta})^T. \quad (2.13) \]

The coefficients $\lambda_i$ determine whether the CP violating complex phase $\theta$ has a nonzero value or not. This phase may enforce the baryon production through sphaleron processes in electroweak baryogenesis. The benefit of this scenario over the violation at tree level is the fact that at low temperature the loop corrections disappear. In this way CP violation is turned off at a certain moment in the history of the universe such that one needs not to worry about physical implications of CP violation at the present.

2.4.3 First order phase transition

Since at the time of the electroweak energy scale the expansion of the universe was much smaller than the rate of B violating processes, the out-of-equilibrium scenario cannot account for the departure from thermal equilibrium. The electroweak baryogenesis scenario is therefore based on a strongly first order electroweak phase transition. A second order or continuous crossover will not result in a strong enough departure from equilibrium to lead to sufficient baryon production. In a first order phase transition, the order parameter $\phi$ has a minimum at $\phi = 0$, separated from a local minimum at $\phi = v$ by an energy barrier. As temperature drops, the two minima first become degenerate in energy at the critical temperature $T = T_c$ and at later times the dislocated minimum become the global minimum of the theory. From the critical temperature downwards, the field starts to tunnel to the global minimum and bubbles of true vacuum nucleate. If the temperature
drops enough, large enough bubbles will nucleate to overcome the surface
tension at the bubble wall and will grow till finally all space is filled with
true vacuum and the transition is complete. The departure from thermal
equilibrium happens when the bubble wall is passing a point in space. The
order parameter changes quickly, and so do other fields. When the wall has
passed, it is very important to demand baryon number violating processes
to be turned off at this temperature. Otherwise these processes would, with-
out the CP violation and departure from thermal equilibrium, wash out the
produced asymmetry. This means we have to demand the sphaleron energy
to be sufficiently larger than the temperature of the true vacuum state after
the phase transition.

Electroweak barygenesis in the standard model is possible in two classes:
1) In local baryogenesis baryons are produced at the bubble wall where B
and CP violating processes occur simultaneously; 2) In nonlocal baryogen-
esis CP violating processes generate in the bubble wall an asymmetry in a
certain quantum number which is carried away from the wall into the true
vacuum. B violation processes the asymmetry in an excess of baryon num-
ber. Although both processes occur in most theories, the nonlocal processes
are always more efficient. The efficiency of both mechanisms are, although
in a different manner, dependent on the speed of the bubble wall. Since in
the local case all processes take place within the wall, the thickness of the
wall and the speed at which it passes a point in space are the determin-
ating parameters. As a short example of a nonlocal scenario consider quarks
and leptons interacting with the wall. As particles are injected through the
bubble wall, they experience the phase transition giving the Higgs field an
expectation value. This means the particles gain mass while injected into
the interior of the bubble, forcing the impulse to decrease. When including
spin, an extra term is added to the resulting force. Due to the CP violation,
particles with opposite helicity will interact with a different resulting force.
This will thus result in a net axial vector current into the true vacuum car-
ried by these particles. The main contribution will come from top quarks
and the tau-lepton, since those Yukawa couplings are the strongest. The ax-
ial asymmetry is inside the bubble converted to a baryon asymmetry. Since
the effect is dependent on the rate of change of the particle masses, a thinner
wall will enhance the mechanism. In this case the problem remains that the
CP violation in the Minimal Standard Model (MSM) without extensions is
not sufficient to reach the required asymmetry. Besides that, the demand
of the sphaleron energy requires the W-boson mass and hence the Higgs ex-
pectation value to be very large immediately after the phase transition. In
the MSM the single Higgs doublet cannot provide such a large expectation
value within the current experimental bounds on the Higgs mass. Moreover,
it is known that when $m_H > 72 \pm 2 GeV$ the transition is crossover, while a
strong first order transition is needed.
As explained in section 2.4.2, the Minimal Supersymmetric Standard Model contains new sources of CP violations. As an example we present here the chargino mass mixing terms [41],

\[
\tilde{\psi}_R M \psi_L = \left( \tilde{\psi}^+ R, \tilde{\psi}^- \right)_R \begin{pmatrix}
m & g H_2(x) \\
g H_1(x) & \mu
\end{pmatrix} \left( \tilde{\psi}^+ L, \tilde{\psi}^- \right)_L + h.c.
\]  

(2.14)

where \( \tilde{\psi} \) and \( \tilde{h} \) are the super partners of the W-boson and the charged Higgs respectively and \( m \) and \( \mu \) are complex mass parameters. The relative phase between \( m \) and \( \mu \) is the CP violating phase, since by field redefinitions one of the two can be rotated to the real axis. Although this provides a stronger violation of CP, it still turns out to be hard to produce the necessary asymmetry. To maximize production, one has to maximize the relative phase giving stronger CP violation and make sure the masses \( m \approx \mu \) are almost equal. Only a very narrow band gives a peak in the production while small deviations ruin the process, making the mechanism subject of fine-tuning. As pointed out, also the thickness of the wall plays an important rôle. This feature is associated with the strength of the first order phase transition, which has to be very strong (the change in the order parameter needs to be of the order of the temperature). In order to achieve this, a very light stop (super partner of the top quark) will be needed, \( m_{\text{st}} < 120 \) GeV. Due to finite temperature loop effects this will change the scalar potential in such a way a stronger phase transition is possible. The Higgs mass still has to be sufficiently small as well (\(< 120 \) GeV) to prevent wash out from unwanted sphaleron processes after the baryon asymmetry is produced. These values are just above the current experimental bounds. Within a couple of years several new experiments at CERN will be initiated when the Large Hadron Collider is taken into use. The new accelerator will be able to detect particles up to energies of one TeV. These experiments will either discover the Higgs particle and/or supersymmetric partners like the stop or push the bounds towards very high energies. If the latter is the case, several models of electroweak baryogenesis will be ruled out since no sufficient enough phase transition can be achieved. In the next-to-minimal supersymmetric model another Higgs singlet is added to the theory, providing a stronger phase transition and new sources for CP violation. It turns out that baryogenesis is much less restricted in such a model [42].

2.5 Affleck-Dine Mechanism

Another mechanism to generate baryon number was proposed by I. Affleck and M. Dine in their 1984 article [43] and was dubbed Affleck-Dine (AD) mechanism. It received a lot of attention the last two decades as an alternative to conventional mechanisms. It relies on supersymmetric grand
unification theories, in which all fermionic fields have a bosonic super partner (see also appendix A). The fields of interest are the scalar squark and slepton fields, super partners of respectively the fermionic spin quark and lepton fields. In super symmetric extensions of the Standard Model like the MSSM the potentials of these fields have certain 'flat directions' (appendix A.3). These are directions in field space in which the potential vanishes. The fields behave therefore as free massless fields and can easily acquire large expectation values during inflation. This is called a coherent field and can carry a large amount of a certain quantum number. Some flat directions carry baryon and lepton number and we are especially interested in directions that carry B-L. In this case, at later time sphaleron processes that violate B+L will not erase the symmetry.

2.5.1 The mechanism in a toy model

To show how the mechanism works consider a complex field \( \phi \) with lagrangian,

\[
L = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi^* - \frac{1}{2} m^2 \phi \phi^* .
\]

(2.15)

This lagrangian possesses a CP symmetry \( \phi \leftrightarrow \phi^* \) and has a global phase symmetry \( \phi \rightarrow e^{i\theta} \phi \) associated with a conserved Noether current,

\[
J_B^\mu = \frac{i}{2} (\phi^* \partial^\mu \phi - \phi \partial^\mu \phi^* ) .
\]

(2.16)

This current can be associated with a conserved quantity. Considering the field \( \phi \) to couple to baryonic matter in a baryon number violating way, we can identify this quantity with a baryon number current. Considering the homogeneous case, the lagrangian represents a harmonic oscillator. In super symmetric models, higher order terms are expected to break the symmetry in a way similar to adding CP violating terms to the Lagrangian. This corresponds to adding anharmonic terms in the harmonic oscillator picture. The rotational invariance is then broken and the system will obtain a nonzero angular momentum. When the system is damped, the amplitude will decrease and the field will spiral to zero, thereby producing a winding number corresponding to a net baryon number production. To demonstrate this we add a potential of quartic interactions to the lagrangian of the form,

\[
L_{int} = \lambda (\phi \phi^*)^2 + \chi_1 \phi^3 \phi^* + \chi_2 \phi^4 + c.c.
\]

(2.17)

The first term still conserves the symmetry and is therefore not of our interest. The last two interactions however clearly violate B and CP for any complex coefficient \( \chi_i \). The coefficients will turn out to be very small and to produce a large amount of baryon number the fields will have to be very large at a certain stage of their evolution. Since at very early times \( H \gg m \), the field behaves massless and it is reasonable to suppose the field has a
large initial expectation value. The cosmological evolution of the field with potential \( V(\phi, \phi^*) \) is then described by the equation of motion,

\[
\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V(\phi, \phi^*)}{\partial \phi^*} = 0 .
\] (2.18)

Here \( H = \dot{a}/a \) denotes the Hubble constant with \( a \) the scale factor. When still \( H > m \), the field is highly overdamped and frozen at its initial value \( \phi_0 \). Only when \( H \) drops well below the mass of the field, it starts to oscillate and the solution in case of no potential in radiation \( (H = 1/2t) \) and matter era \( (H = 2/3t) \) respectively reads,

\[
\phi = \frac{\phi_0}{(mt)^k} \sin(mt) \] (2.19)

Here \( k = 1 \) during matter era and \( k = 3/2 \) during radiation era. Since in both cases the energy scales with the scale factor cubed, the oscillating field can be thought of as a coherent state of pressureless particles. Taking into account the quartic coupling terms and considering the initial field strength \( \phi_0 \) to be real, the imaginary part of the equation of motion for small coefficients \( \chi_i \) can be written,

\[
\ddot{\phi}_i + 3H\dot{\phi}_i + m^2\phi_i \approx Im [\chi_1 + \chi_2] \phi_3^3 .
\] (2.20)

The righthand side will decay in time similar to the harmonic oscillator example described above, resulting in a spiraling field in the complex plane. The late time solution then yields,

\[
\phi_i = b Im [\chi_1 + \chi_2] \phi_3^3 \frac{\phi_0^4}{m^2(mt)^n} \sin(mt + \alpha) .
\] (2.21)

Here \( b \) and \( \alpha \) are constants of order of unity, both different for matter and radiation era. Furthermore, \( n = 1 \) \( (3/4) \) in matter \( (\text{radiation}) \) era. Plugging these results in the definition for the current, for late times the baryon number asymptotically goes to,

\[
n_B = \frac{2bIm [\chi_1 + \chi_2] \phi_0^4}{m(mt)^2} \sin(\alpha) .
\] (2.22)

This is a constant in a comoving volume, as it should be. The constants \( b \) and \( \alpha \) are numerically calculated.

### 2.5.2 AD mechanism with supersymmetry

Taking this example to a more realistic model, we can take squark and slepton fields to be the scalar fields that carry baryon and lepton number. The flat directions which carry these quantum numbers can provide the
small coefficients of the potential because of which this model works. An example of a flat direction carrying $B - L = -1$ is given by [44],

$$Q_1^\alpha = \frac{1}{\sqrt{3}} \begin{pmatrix} \phi \\ 0 \end{pmatrix}, \quad L_1 = \frac{1}{\sqrt{3}} \begin{pmatrix} 0 \\ \phi \end{pmatrix}, \quad \bar{d}_2^\alpha = \frac{1}{\sqrt{3}} \phi.$$  \hspace{1cm} (2.23)

Here denotes $\phi$ the complex scalar of the superfield parameterizing the flat direction, the superscripts are color indices and the subscript generation indices. All flat directions in the MSSM are most recently cataloged by Gherghetta, Kolda and Martin [45]. The product of the $m$ fields making up the flat direction are often combined to the invariant operator $X$. In the example above $X = Q_1 L_1 \bar{d}_2 = c \phi^m$ with $m = 3$. The flat direction can be lifted by soft supersymmetry breaking and non-renormalizable terms in the superpotential. The latter ones appear in two types. The invariant $X$ can appear to some positive power $k$,

$$W = \frac{\lambda}{nM^{n-3}} X^k = \frac{\lambda}{nM^{n-3}} \phi^n,$$  \hspace{1cm} (2.24)

in which $n = mk$ and $M$ a large mass scale like the GUT or Planck scale. Depending on $X$ being even or odd under R-parity $k$ has a minimum of 1 respectively 2 (see appendix A.3). Another possible type of terms contains a number of fields making up the flat direction and one other field,

$$W = \frac{\lambda}{M^{n-3}} \psi \phi^{n-1}.$$  \hspace{1cm} (2.25)

Both type of terms give the same lowest order contribution to the potential, which dominates the potential for sufficient large values of the field,

$$V(\phi) = \frac{|\lambda|^2}{M^{2n-6}} (\phi \phi^*)^{n-1}.$$  \hspace{1cm} (2.26)

This is clearly conserving the U(1) quantum number carried by the flat direction and therefore less interesting. The other possibility of lifting the potential comes from soft symmetry breaking. The general form of these are given by mass terms and interaction terms like,

$$V(\phi) = \mu^2 \phi \phi^* + \frac{A}{M^{m-3}} \phi^m.$$  \hspace{1cm} (2.27)

In the present universe with broken super symmetry, $\mu$ and $A$ are of the order of the weak scale $m_{3/2}$, the gravitino mass. In the early universe, however, supersymmetry is broken by the finite energy density, resulting the parameters to be of the order of the Hubble constant $\mu \sim A \sim H$. Furthermore, the interaction term in eqn. (2.27) violates the U(1) symmetry of the flat direction and has in general a complex coefficient $A$. Considering a flat direction carrying $B - L$, this term gives both a source for $B - L$
violation and CP violation. The toy model pointed out above can then be applied to calculate the resulting $B - L$ asymmetry. After the squark and slepton condensates have acquired a large baryon and/or lepton number, this is transported to ordinary matter by scattering effects as soon as the temperature of the universe drops sufficiently.

2.6 Leptogenesis

As explained in section 2.4.1, sphaleron processes in the Weinberg-Salam theory of weak interactions violate $B + L$ but conserve $B - L$ at sufficiently high temperatures. This led Fukugita and Yanagida [46] to the realization that instead of baryon number it could be sufficient to produce net lepton number in the early universe. During reheating, the temperature of the universe increases to a scale at which sphalerons can process the lepton number to a baryon asymmetry. The important assumption made in order to produce lepton number is the presence of neutrino mass and the existence of right handed neutrinos. Although neutrinos are often assumed to be massless, indirect evidence for nonzero masses has become more extended recently [47]. No direct evidence exists yet and the upper boundary for neutron masses is set to the low value of 0.68$eV$ [24]. Till now, right handed neutrino’s are not seen in experiments, indicating a very large mass if they exist. In the Standard Model there is no profound reason for the neutrino’s not to have a mass like in case of the photon. At the level of renormalizable terms it is however not possible to implement it in the model. This means again new physics seems to be needed to explain the observed solar and atmospheric neutrino oscillations, indicating nonzero mass.

2.6.1 See-saw mechanism

An elegant way to explain the light left handed neutrino masses and heavy right handed neutrino’s is provided by the see-saw mechanism [48]. This mechanism can be embedded in a SO(10) unification theory of the Standard Model. Originally, no Dirac mass term for neutrino’s can be present in the lagrangian due to the absence of their right handed companions. A normal (left handed) four component Dirac spinor $\psi_L$, like the electron field, describes both $e_L^-$ and $e_R^+$. This means, if a particle respects parity conservation, $\psi_L$ as well as $\psi_R$ is needed to give a complete description of the interactions. In the case of neutrinos however, only $\nu_L$ is needed since $\nu_R$ and $\bar{\nu}_R$ are sufficient to explain the parity violating weak interactions. Since for a Majorana spinor the particle equals its antiparticle, it is possible to add a Majorana mass term of the form,

$$m_L \bar{\psi} \psi = m_L (\bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R) \quad (2.28)$$
2.6. LEPTOGENESIS

Since this mass term violates lepton number by 2 units, the Standard Model and GUTs based on SU(5) cannot contain them and possess therefore massless neutrinos. A Dirac mass term in SU(5) when including right handed neutrinos cannot explain the smallness of the mass. If B-L is violated, as is the fact in SO(10) GUT’s, a Majorana term like in eqn. (2.28) can be added for a left and right handed neutrino. Since the $\nu_R$ is a singlet, it fits well in the SO(10) model, having an additional 1 besides the $\overline{5}$ and 10 representations of section 2.3.1. The fact that $\nu_R$ is a singlet also gives the property that its mass $m_R$ can be unrestricted large. The $\nu_L$ are doublets, and obtain their Majorana masses by the non renormalizable Yukawa coupling,

$$L_{\nu M} = \frac{1}{M} (LH)^2,$$

in which $L$ denotes the left handed lepton doublet and $H$ the Higgs doublet. The cut off mass $M \sim 10^{14} GeV$ is the scale associated with B-L violation and we can identify it with the right handed neutrino mass. Taking the expectation value of the Higgs field to be roughly $v = m \sim M_W$, all the mass terms can be collected in the mass matrix,

$$ (\nu_L \; \nu_R^c) \begin{pmatrix} m_L \sim \frac{m^2}{M} & \frac{1}{2} m_D \sim m \\ \frac{1}{2} m_D \sim m & m_R \sim M \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix} + h.c. \tag{2.30} $$

The eigenvalues of this matrix give the neutrino masses $m_1 \sim \frac{m^2}{M}$ and $m_2 \sim M$. The exact values of $m$ and $M$ are yet to be determined, but more important is the fact that increasing $M$ gives decreasing $m_1$ and increasing $m_2$. With the choices for $m$ and $M$ as made above this provides a solid model for explaining the discrepancy between the left and right handed neutrino masses within the experimental bounds.

2.6.2 Lepton number generation

As pointed out in the previous section, lepton number is violated in the couplings of $\nu_R$ and CP violation can be present in the Yukawa couplings. As a result, the heavy $\nu_R$ can decay with different partial widths to both $H + \nu_L$ and $H + \bar{\nu}_R$. However, similar to the case in GUT baryogenesis (section 2.3), the CP violating phase cancels out on tree level. When considering the one loop diagrams of the decay of the heaviest $\nu_R$, the exchanged $\nu_R$ can be on shell. As a result, CP violation truly enters and the decay widths are different. The asymmetry is proportional to the Yukawa coupling $y_{\nu}$,

$$\epsilon = \frac{\Gamma(\nu_R \rightarrow lH) - \Gamma(\nu_R \rightarrow \overline{hh})}{\Gamma(\nu_R \rightarrow lH) + \Gamma(\nu_R \rightarrow \overline{hh})} = \frac{1}{8\pi|y_{\nu}|^2} \sum_{i=2,3} Im \left| y_{\nu} \right|^2 \left( \frac{M^2}{M_W^2} \right). \tag{2.31}$$
Here \( l \) can be any lepton and \( \nu_R \) is assumed to be the heaviest right handed neutrino. \( M_H \) and \( M \) are the masses of the Higgs and the \( \nu_R \) respectively and the function \( f(x) \) accounts for radiative corrections, given by,

\[
f(x) = \sqrt{x} \left[ \frac{x - 2}{x - 1} + (x + 1) \log(1 + 1/x) \right] \quad \text{Standard Model, } \quad (2.32a)
\]

\[
= \sqrt{x} \left\{ \frac{2}{x - 1} + \log(1 + 1/x) \right\} \quad \text{MSSM. } \quad (2.32b)
\]

The evolution proceeds in a similar way as in GUT baryogenesis, with out of equilibrium decay. Only in this case, only leptons are produced and no baryons, hence producing a net B-L current. Sphalerons process this asymmetry to a separate B and L asymmetry. Since temperatures are well above the particle masses, this calculation can be done by use of simple thermodynamics, assuming all particles to be massless. From relations between the chemical potentials of all particles of interest it is obtained [49],

\[
B_f = \frac{8N + 4(m + 2)}{24N + 13(m + 2)} (B - L)_i. \quad (2.33)
\]

Here denotes \( N \) the number of particle species and \( m \) the number of Higgs doublets. For reasonable parameters this gives an asymmetry of the desired magnitude. The problem in this mechanism is the large value of \( M \), which is far out of reach of any accelerator ever build. Either a very persuasive model of neutrinos or indirect evidence of the processes involved is required to exclude or confirm this model.

### 2.7 Gravitational baryogenesis

The model for baryogenesis as investigated in the next chapter is inspired by the idea proposed in the 2004 article of Davoudiasl et al [50]. This kind of mechanism is proposed first by Cohen and Kaplan [51], they stated that baryogenesis can be realized through a derivative coupling of the baryon number current \( J_B^\mu \) to a scalar field. Also Balaji and Brandenberger considered the possibilities of baryogenesis with a single scalar field [52, 53]. Davoudasl et al altered the model of Cohen and Kaplan to a derivative coupling to curvature, something they dubbed gravitational baryogenesis. Such a coupling looks like,

\[
\frac{1}{M_*^4} \int d^4x \sqrt{-\bar{g}} (\nabla_\mu R) J_B^\mu. \quad (2.34)
\]

Here \( M_* \) is a cut-off scale of the theory and \( R \) the Ricci scalar of curvature. In fact, any orthogonal current leading to generation of B-L charge will do, such that the asymmetry will not decay through sphaleron interactions.
The interaction of eqn. (2.34) violates CP but is CPT conserving in vacuum. However, in an expanding universe the derivative of the Ricci scalar can be nonzero and dynamically violate CPT. This will shift the energy of the particles and anti particles differently and hence generate an asymmetry of particle production in thermal equilibrium. Departure from thermal equilibrium as stated in the third condition of Sakharov is not needed anymore. To create B violation and thus particles we consider B violating processes in thermal equilibrium which decouple at a temperature \( T_B \). Due to the difference in energy contribution to particles and anti particles the chemical potentials are changed different with \( \mu \sim \pm \dot{R}/M^2 \). The B violating processes will therefore push the universe to a state of B asymmetry. As the universe expands and cools, temperature will drop below \( T_B \) and the asymmetry freezes out. The remaining asymmetry is then given by \[ n_B \approx \dot{R} \left| M^2 T \right| \text{ at } T = T_B. \] (2.35)

With \( s \) the entropy density. A few problems arise in this model though. First of all, the Ricci scalar will be zero during radiation era and constant during inflation. As a result this mechanism can only work during matter era. At that time, all known baryon number violating processes are out of equilibrium and the mechanism fails. This can be seen in another way by rewriting eqn. (2.34) to a total derivative and demand it to vanish at the boundaries, 

\[ \frac{1}{M^2} \int d^4 x \sqrt{-g} R (\nabla \mu J_B) \] . \hspace{1cm} (2.36)

This term decays as \( a^{-4} \) with \( a \) the scale factor, and will not be capable to produce a net baryon current. A possibility is the implementation of a scalar-tensor theory of gravity, which will lead to a phase of inflation with decaying curvature \( R \propto t^{-2} \). This seemed not to be the intent of the authors however. In the last chapter we will assume Brans-Dicke theory and generalize the theory to include a complex scalar field. As we will see, in this case curvature coupling will lead to baryon production.
Chapter 3

Scalar fields

Although still no empirical evidence is available, scalar fields have played an important rôle in particle physics and cosmology throughout the history. The driving particle behind inflation, the inflaton, is supposed to be a scalar field, as well as symmetry breaking order parameters like the Higgs field. Also in some models for baryogenesis, like the Affleck-Dine mechanism, scalar fields are involved, since they can carry in a natural way baryon and lepton number. In the next chapter we will construct a model for baryogenesis involving scalar fields. Therefore we first study in a very general way the properties of a complex scalar field $\phi$ in the homogeneous case with a CP violating potential. We can introduce a very general potential $V(\phi)$ containing CP violating terms in the following way,

$$V(\phi) = V_2 + V_3 + V_4,$$

where

$$V_2 = -\frac{1}{2}m^2|\phi|^2 + \frac{1}{2}\mu^2\phi^2 + c.c.,$$

$$V_3 = \frac{1}{3!}\sigma_1\phi^3 + \frac{1}{3!}\sigma_2\phi^2\phi^* + c.c.,$$

$$V_4 = \frac{1}{4!}\lambda_0|\phi|^4 + \frac{1}{4!}\lambda_1\phi^4 + \frac{1}{4!}\lambda_2\phi^3\phi^* + c.c.$$

We start with a general equation of motion on a homogeneous background with Friedman-Robertson-Walker metric and Hubble rate $H = \dot{a}/a$ with $a$ the scale factor. Using the canonical form of the kinetic term, we can write,

$$\Box \phi + \frac{\partial V}{\partial \phi^*} = \ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi^*} = 0 . \quad (3.2)$$

We can introduce a scaled field $\varphi$ by $\phi = a^{-w}\varphi$, the scaling $w$ depending on the era under consideration. Substituting this in the equation of motion and multiplying by $a^w$ gives,

$$\ddot{\varphi} + \left[ (3 - 2w)\frac{\dot{a}}{a} \right] \dot{\varphi} + w \left[ (w - 2) \left( \frac{\dot{a}}{a} \right)^2 - \frac{\ddot{a}}{a} \right] \varphi + a^w \frac{\partial V}{\partial \phi^*} = 0 . \quad (3.3)$$
CHAPTER 3. SCALAR FIELDS

When considering the scaling $w = 3/2$ we see the friction term drops out. This equation can be multiplied by $\varphi^*$ and subtracted from the equation of motion of the conjugate field multiplied by $\varphi$ to obtain,

$$\varphi \dot{\varphi} - \varphi^* \dot{\varphi} = \frac{d}{dt}(\varphi \varphi^* - \varphi^* \varphi) = a^{3/2} \left( \varphi^* \frac{\partial}{\partial \varphi^*} - \varphi \frac{\partial}{\partial \varphi} \right) V . \quad (3.4)$$

We can assume that this scalar field carries a charge $Q$ to be specified later. Associated with this charge is a current $J_\varphi$ containing the charge density and flow. With the previous definitions and the definition of the first element of the scalar current $J_\varphi$ it is easily seen this yields the following relation between $J_\varphi$ and the potential,

$$\frac{d}{dt}(a^3 J_\varphi) = \frac{i}{2} a^3 \left( \phi^* \frac{\partial}{\partial \phi^*} - \phi \frac{\partial}{\partial \phi} \right) V . \quad (3.5)$$

The current, nothing less than the charge density, is clearly sourced by the CP violating part of the potential, since this is exactly what the right hand side comes down to. We will call this $S$. Since the zeroth component of the current in this homogeneous case is in fact the scalar particle density $n_\varphi$, and the entropy $s$ scales such that $a^3 s = const$ we can write eqn. (3.5) in the even more lucid form,

$$\frac{d}{dt} \left( \frac{n_\varphi}{s} \right) = \frac{S}{a^3 s} . \quad (3.6)$$

Based on these derivations, in which no choice of era and hence scale factor was yet made, we can define what conditions are best for baryogenesis. The best thing to have is a growing source, since this would enhance the baryon to entropy ratio in a progressive manner. The only way this can be achieved is with a $V_2$ type potential with radiation era scaling, since in this case $S \sim a$. The mass term must be sub dominant, so we have to demand a small mass compared to the Hubble rate: $m^2 \varphi \ll 3H \dot{\varphi}$. A constant source would do well too and can be achieved with a $V_3$ potential in radiation era and $V_2$ potential in matter era. When $S$ is decaying, the baryon to entropy ratio will be dependent on initial conditions which is not preferable. This is the case in the Affleck-Dine mechanism for instance. Although an argument why the field should be large at some initial time is presented, it remains questionable whether one should prefer a mechanism depending on it. It somehow seems more natural to really produce something, depending on the late time behavior. Let us as an example consider a constant source during matter era, implying a $V_2$ potential. During matter era the scale factor scales as $a = t^{2/3}$ and $S_2$, the source due to the potential $V_2$, reads,

$$S_2 = a^3 Im(\mu^2 \varphi^2) = Im(\mu^2 \varphi^2) . \quad (3.7)$$
To solve this, we first need to solve the equation of motion,

$$\ddot{\varphi} + m^2 \varphi + \mu^2 \varphi^* = 0,$$

(3.8)

by making the ansatz of $$\varphi = \varphi_0 e^{\pm i\xi t}$$. We are actually cheating here, since in this toy model the complex phase in the lagrangian can actually be removed by a simple field redefinition. For now we will neglect this to be able to study the effect of the complex phase. Substituting the ansatz in both the equation of motion of the field and its conjugate gives two coupled equations for $$\phi$$ and its conjugate, which can be solved by putting the determinant of the mixing matrix to zero. From this condition we obtain the oscillatory solutions,

$$\varphi = \varphi_+^0 e^{\pm i\xi t}, \quad \xi = \sqrt{m^2 + |\mu|^2}$$

(3.9)

where we can write the constants $$\varphi_+^0 = |\varphi_+^0| e^{\pm i\theta_\pm}$$ in terms of their complex phase $$\theta_\pm$$. Writing also the coefficient $$\mu^2 = |\mu|^2 e^{2i\theta_\mu}$$ in terms of its complex phase we arrive after some algebra at,

$$\text{Im}(\mu^2 \varphi^2) = |\mu|^2 \left[ |\varphi_+^0|^2 \sin(2(\xi t + \theta_+ + \theta_\mu)) + |\varphi_-^0|^2 \sin(2(\xi t - \theta_- - \theta_\mu)) + |\varphi_+^0||\varphi_-^0| \sin(\theta_+ + \theta_- + 2\theta_\mu) \right].$$

(3.10)

This oscillating expression is to be considered during a period of time much longer than the period. Taking the time average we arrive at,

$$\langle \text{Im}(\mu^2 \varphi^2) \rangle_{\text{time}} = |\mu|^2 |\varphi_+^0 \varphi_-^0| \sin(\theta_+ + \theta_- + 2\theta_\mu).$$

(3.11)

From eqn. (3.6) we can write down the baryon to entropy ratio in this case to be,

$$\frac{n_\phi}{s} = \int_{t_i}^{t} \frac{\text{Im}(\mu^2 \varphi^2)}{a^3 s} = \frac{1}{a^3 s} |\mu|^2 |\varphi_+^0 \varphi_-^0| \sin(\theta_+ + \theta_- + 2\theta_\mu)(t - t_i).$$

(3.12)

This clearly leads to a linear growth of the ratio not dependent on the initial time. The complex phase $$\theta_\mu$$ of the potential can be shifted away and is therefore not physical. The other two phases will remain in that case and correspond to a nontrivial initial condition which can be dubbed a kick. The initial field rotates in the complex plane which generates the condensate.

### 3.1 Inflation

When considering inflation, eqn. (3.2) can be solved and will lead to an exponential solution. Without including any of the potentials, the massless case, the equation of motion yields,

$$\Box \phi + \ddot{\phi} + 3H \dot{\phi} = 0.$$  

(3.13)
Here $H$ denotes the Hubble constant during inflation. This equation can easily be solved, giving,

$$\phi = \phi_0 e^{-3Ht} = \frac{\phi_0}{a^3},$$

with $a = e^{Ht}$ the scale factor during inflation. We can see a field of this type completely dilutes in a background of exponentially inflating space. This effect was one of the desired features of inflation, since in a similar way it flattens out all unwanted inhomogeneities of the pre-inflationary universe. In the case of a massive scalar field we have the massive Klein-Gordon equation,

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0.$$  

To solve this we can make an exponential ansatz and solve for the exponent. The solution is given by,

$$\phi = \phi_0 e^{\pm k \cdot t} + \phi_0 e^{\pm k \cdot t}$$

$$k_{\pm} = -\frac{3}{2}H \left(1 \pm \sqrt{1 - \frac{2m^2}{9H^2}}\right).$$

with $\phi_0$ constants of integration. The only possible way of this to become a growing solution is when the mass term becomes negative, implying an imaginary mass. This results in an energy instability in the theory, but such terms naturally enter the theory when including CP violating potentials. In the next chapter we will consider such a model and also assume the mass squared to be replaced by the curvature. This provides a natural reason for the field to be on top of the instability (local maximum) at the beginning of inflation, since in a radiation dominated pre-inflationary era the curvature is zero. The expression for the current as stated in eqn. (3.6) is still valid during inflation, but since we will use another approach in the rest of the thesis we will derive it in a different way. The same line of thought as the previous section can be followed, although we will now consider eqn. (3.2) as a start. Writing the complex field in its real and imaginary component as $\phi = \phi_+ + i\phi_-$, the equation of motion can be recast to,

$$\Box \phi_{\pm} = \frac{1}{2} \left( \frac{\partial}{\partial \phi} \mp \frac{\partial}{i \partial \phi^*} \right) V(\phi, \phi^*) \quad (3.17)$$

$$= \frac{1}{2} \frac{\partial}{\partial \phi_{\pm}} V(\phi_+, \phi_-).$$

In the general potential of eqn. (3.1), there will be a strong mixing between the two fields $\phi_{\pm}$. In analogy to the calculations above, an expression for the current $J_\phi$ can be written,

$$\frac{d}{dt}(a^3 J_\phi) = a^3 \left( \phi_+ \frac{d}{d\phi_-} - \phi_- \frac{d}{d\phi_+} \right) V(\phi_+, \phi_-).$$
It is easily verified that again terms of the potential without CP violation cancel out, as is to be expected. Since we are considering the homogeneous case we can again write this result in terms of the baryon to entropy ratio, assuming $a^3 s$ to be constant and denoting the right hand side of eqn. (3.18) by $D$,

$$\frac{d}{dt} \left( \frac{n_\phi}{s} \right) = \frac{D}{a^3 s}. \quad (3.19) $$

It is easy to convince oneself that indeed $D = C$. We will postpone the derivation of the solution of $\phi_\pm$ and the current till the next chapter where we will consider a model in which this type of situation is present.

## 3.2 CP violation

To show why the presence of complex phases is so important for baryogenesis, consider an equation of motion similar to the one presented in eqn. (3.8),

$$\Box \phi + \alpha^* \phi^* + V'(\phi) = 0, \quad (3.20)$$

where $\alpha = |\alpha|e^{i\varphi_\alpha}$ denotes a coefficient with a complex phase $\varphi_\alpha$, irremovable due to the potential $V(\phi)$. Under the CP operator the field changes $(CP)\phi(x) \to \phi^*(\tilde{x})$, with $\tilde{x} = -\vec{x}$ and $\tilde{x}^0 = x^0$, but leaves the coefficient unchanged. The equation of motion of the complex field after CP acted on it then reads,

$$\Box \phi + \alpha \phi^* + V'(\phi) = 0, \quad (3.21)$$

which is clearly not the same. Note that the d'Alembertian $\Box$ will only be invariant under CP for certain space-times. This is the case for a de Sitter space as defined, though. As $\varphi_\alpha$ grows, the imaginary part of the exponential $(\sin \varphi_\alpha)$ gets a different sign for the two equations. Hence the CP symmetry is broken by introducing the complex coefficient. For small phases, the larger $\varphi_\alpha$ is, the larger the difference between eqn. (3.20) and eqn. (3.21) becomes. The conclusion is that a complex phase introduces a difference in dynamical behavior of the imaginary part of the field corresponding to CP violation. This is the second criteria of Sakharov needed for baryogenesis.

In most of the thesis we will use the formal definition of a scalar current (the first element),

$$J_\phi = \frac{i}{\sqrt{2}} (\phi \dot{\phi}^* - \phi^* \dot{\phi}), \quad (3.22)$$

in which we used the decomposition of the field in its real and imaginary part $\phi = \phi_+ + i\phi_-$. From the first definition it is clear that we are interested
in the CP odd part of the field. This is the part where the equations of motion deviate as shown in the previous paragraph. Moreover, from the second definition we can see that the real and imaginary part of the field need to have a different time evolution. Once these conditions are satisfied and $\phi$ grows fast enough in time, a growing solution for $J_\phi$ is obtained. In the next chapter we will construct a model in which this is the case.
Chapter 4

Baryogenesis in Brans-Dicke theory

4.1 Introduction

So far we have explored some of the features of Brans-Dicke (BD) theory and the attempts made in the past to model the creation of matter in the early universe. Many of the described models still suffer from inconsistencies of constraints, boundary problems or remain unproven, though. The fact no consensus is yet reached leaves the door open for the search for new possibilities. The appearance of a new scalar field in BD theory gives opportunities for baryogenesis not addressed before. Gravitational baryogenesis as pointed out in section 2.7 uses a coupling to curvature also present in BD theory and has a dynamical character. We will rather make a generalization of the theory, changing the scalar to a complex one. The field can then carry a certain charge $Q$ proportional to $B$ and $L$. CP violating interactions, also coupled to curvature through the Ricci scalar, will be included to drive the mechanism. The original BD theory, apart from the interactions, is reobtained in one certain field configuration. As we will see, we can derive from this theory a growing scalar current expressing the charge density $Q$ as shown in the previous chapter. The attractive feature of this theory is its simplicity, describing baryon production with only the help of the BD scalar. It is also very elegant to picture the field responsible for the strength of the coupling of matter to gravity also responsible for the generation of matter itself during inflation. In General Relativity such a mechanism is not possible since the metric cannot contain a CP violating phase, but when including the BD scalar, CP violation can be combined with a theory of gravity. As will be shown, this results in a mechanism which can produce a baryonic current even during the era of inflation when space expands very rapidly. The choice of this era was made when including the curvature coupling. During matter domination the universe expands sub luminal and the
CHAPTER 4. BARYOGENESIS IN BRANS-DICKE THEORY

Ricci scalar will fall of as $1/t^2$, which will suppress the interactions and thus any production of baryon number. Since $R = 0$ during radiation era we are forced to investigate the theories properties during inflation. The departure from thermal equilibrium is again dynamically broken by the expansion of the universe.

4.2 Generalization of the theory

For convenience we will once more display the Brans-Dicke action, now in natural units in which $c = G_0 = h = 1$,

$$S = 16\pi \int d^4x(-g)^{1/2} \left[ \Phi R - \frac{\omega}{\Phi} (\partial_\mu \Phi \partial^\mu \Phi) g^{\mu\nu} \right] + \int d^4x(-g)^{1/2} L . \quad (4.1)$$

Here we denote by $R$ the Ricci scalar curvature, by $L$ the ordinary Lagrangian of matter and non-gravitational fields and $\omega$ is the coupling constant. As pointed out in the section on extended inflation, inflationary expansion in a Brans-Dicke universe is slowed down to powerlaw. This results in a decaying curvature of $R \propto t^{-2}$. The solutions for the scalar $\Phi$ and corresponding scale factor in the original BD theory of eqn. (4.1) are,

$$\Phi = M_p^2 (1 + \chi t/\alpha)^2$$
$$a(t) = a_0 (1 + \chi t/\alpha)^{\omega+1/2} . \quad (4.2)$$

Here denotes $M_p$ the Planck mass, $\chi^2 = 8\pi \rho_f / 3M_p^2$ the Hubble constant squared in the Einstein theory of gravity and $\alpha^2 = (3 + 2\omega)(5 + 6\omega)$. As can be seen, for early times and large $\omega$ the expansion will be near exponential $a(t) \approx e^{\chi t}$ and $R$ only very slowly decaying. We will assume this is the case in our theory and hence the de Sitter metric of an exponential expanding universe can be used,

$$g_{\mu\nu} = \text{diag}(-1,a^2,a^2,a^2); \quad a(t) = e^{Ht} , \quad (4.3)$$

with $\chi = H$ the constant Hubble parameter. We are interested in the production of a large condensate of a charge $Q$, proportional to both baryon number $B$ and lepton number $L$, given by,

$$Q = q_B B + q_L L . \quad (4.4)$$

We therefore demand the BD scalar to be complex and charges under $Q$. We then have to change the Lagrangian such that it contains the introduced complex scalar field. In order to satisfy the second Sakharov condition we will have to add CP-violating terms such that all three conditions as proposed by Sakharov are fulfilled. For dimensional reasons as well for convenience the substitution $\Phi \rightarrow \phi \phi$ is sometimes made in the literature,
4.2. GENERALIZATION OF THE THEORY

as encountered before in this thesis. We will make a likewise substitution, since it changes the kinetic term to the canonical form. When making the field complex, the action has to be modified in order to remain hermitian. We have the choice of either doing this by adding the complex conjugate term,

$$L_{\text{kin}} = -\frac{1}{2} \omega (\partial_\mu \phi \partial_\nu \phi) g^{\mu\nu} - \frac{1}{2} \omega^* (\partial_\mu \phi^* \partial_\nu \phi^*) g^{\mu\nu}, \quad (4.5)$$

with $\omega$ a complex coefficient, or by adding the hermitian form,

$$L_{\text{kin}} = -\omega (\partial_\mu \phi \partial_\nu \phi^*) g^{\mu\nu}. \quad (4.6)$$

In the latter case $\omega$ has to be real. We assume the mass term $m \phi \phi^*$ not to be present in our theory and denote the collection of all interaction terms, coupled to gravity through the Ricci scalar $R$, by $I(\phi, \phi^*)$. When considering CP-violating terms quadratic, cubic and quartic in the scalar field, the general form of $I(\phi, \phi^*)$ is given,

$$I(\phi, \phi^*) = \mu |\phi|^2 + \sigma_1 \frac{3!}{3!} |\phi|^3 + \sigma_2 \frac{3!}{3!} |\phi|^2 \phi^* + \frac{\lambda_1}{4!} |\phi|^4 + \frac{\lambda_2}{4!} |\phi|^3 \phi^* + \text{c.c.} \quad (4.7)$$

The $\sigma_i$ and $\lambda_i$ are all complex coefficients, signalling CP violation. When using the first possible kinetic term, we can split the field in its real and imaginary part and see the two mix. In order to decouple them and solve for them separately, the fields can be rotated in the complex plane to $\phi^+$ and $\phi^-$. Eqn. (4.5) can then be rewritten to,

$$L_{\text{quadr}} = \sum_{\pm} \frac{\pm |\omega|}{|\mu|} (\partial_\mu \phi_{\pm})(\partial_\nu \phi_{\pm}) g^{\mu\nu}. \quad (4.8)$$

It is clear the two fields $\phi^+$ and $\phi^-$ both have a different sign contribution to the stress-energy density, resulting in negative energy states of the $\phi^+$ field.

In the original Brans-Dicke theory the gravitational scalar field contributes positively to the stress-energy density. In the second possible kinetic term no mixing occurs and there is still phase freedom left in this theory. Considering also the quadratic terms, we can make a global phase transformation of the form,

$$\phi \rightarrow e^{i\varphi} \phi. \quad (4.9)$$

This phase change will leave the kinetic term invariant and rotates the coefficients $\mu$ of the quadratic terms to the real axis. We finally rescale the field in order to set these coefficients to unity. Since mixing occurs between the field and its conjugate, it is preferable to solve for the real and imaginary part separately and call these fields $\phi_{\pm}$ again. For the kinetic and quadratic terms we then have,

$$L_{\text{quadr}} = \frac{\omega}{|\mu|} (\partial_\mu \phi_{\pm})(\partial_\nu \phi_{\pm}) g^{\mu\nu} \mp \phi_{\pm}^2 R. \quad (4.10)$$
As can be seen, no imaginary phases occur anymore in this part of the action. Since their presence is needed for baryogenesis, there are two possible ways to proceed. First we can include the higher order terms of $I(\phi, \phi^\ast)$. Even after the phase transformation the coefficients of the cubic and quartic terms still remain complex. The less attractive feature is that the equations of motion cannot be solved analytically anymore and thus simplifications and perturbations are needed. The second possibility is a combination of the two kinetic terms written in eqn. (4.5) and (4.6), such that the energy states are positive but the phase freedom is exhausted and CP violation occurs at the quadratic level. We will consider this case first, but we should first note the following. When transforming the original form of the Brans-Dicke lagrangian into the complex scalar form, we used the fact that in the literature often the substitution $\Phi \rightarrow \phi \phi$ is made in eqn. (1.4). This resulted in the quadratic effective mass terms. The linear single field terms $\kappa \phi + \kappa^\ast \phi^\ast$ could be considered as well, however. These are clearly CP violating and their complex phase will remain complex after the previously performed transformations. The equation of motion including these terms yields,

$$\Box \phi_{\pm} \pm \frac{R}{\omega} \phi_{\pm} \pm \frac{R}{\omega} \kappa_{\pm} = 0.$$  \hspace{1cm} (4.11)

Here the coefficient $|\mu|$ is absorbed in a redefinition of $\omega$, as will be done throughout the rest of the article, and $\pm \kappa_{\pm}$ is just the vector denoting the complex conjugated coefficient $\kappa^\ast$. There is still freedom left to shift the field $\phi_{\pm} \rightarrow \phi_{\pm} - \kappa_{\pm}$ and this translation will shift the phase to zero. When including higher order interaction terms in the lagrangian, as will be done in the next section, removing the linear term is less trivial. The complex phase could be rotated to the real axis by a proper redefinition of the field. Removing the term completely would give the quadratic term again a complex phase. This phase is completely due to the presence of the higher order terms and therefore no new physics occurs in this particular case. With this realization it should be clear these single field terms do not have to be under our consideration, since they cannot lead to additional CP violation.

### 4.3 Double kinetic term

We will now consider the more extended kinetic structure by imposing that both terms (4.5) and (4.6) are present in the theory. The first introduces a complex phase which can be removed by a rotation, the second shifts the kinetic term of $\phi_{\pm}$ such that it becomes negative and assures positive energy states. The benefit of this model is that the phase freedom is exhausted and cannot be used anymore to remove the phase of the quadratic terms. As a result, this is a physical phase and CP violation is present at linear level in
the equation of motion. To see how this works in practice we start with the lagrangian,
\[ L = -\frac{1}{2} \omega (\partial_\mu \phi \partial_\mu \phi) g^{\mu \nu} - \frac{1}{2} \omega^* (\partial_\mu \phi^* \partial_\mu \phi^*) g^{\mu \nu} - \rho (\partial_\mu \phi \partial_\nu \phi^*) g^{\mu \nu} + \mu \phi^2 R + \mu^* \phi^2 R. \] (4.12)

We perform the rotation such that \( \omega \) becomes real and the decomposition in the \( \phi_\pm \) fields reads,
\[ L = - (\rho + \omega) (\partial_\mu \phi_+ \partial_\mu \phi_+) g^{\mu \nu} - (\rho - \omega) (\partial_\mu \phi_- \partial_\mu \phi_-) g^{\mu \nu} + 2 \bar{\mu}_r (\phi_+^2 - \phi_-^2) R - 4 \bar{\mu}_i \phi_+ \phi_- R. \] (4.13)

Here denotes \( \bar{\mu}_{r,i} \) the real and imaginary part of the rotated coefficient \( \mu \) respectively. Due to the rotation the phase information of \( \omega \) is still present in these coefficients. The coefficients of the kinetic terms have to be such that \( \rho - \omega > 0 \) in order to have positive energy states, and \( \omega \) is now real. The presence of CP violation is seen in the mixing term of the two fields, the last term in the expression. On a FRW background this lagrangian results in the equation of motion,
\[ -2(\rho \pm \omega) (\ddot{\phi}_\pm + 3H \dot{\phi}_\pm) \pm 4 \bar{\mu}_r \phi_\pm R - 4 \bar{\mu}_i \phi_\mp R = 0. \] (4.14)

To solve this, we can make an exponential ansatz,
\[ \phi_\pm = \phi_0^\pm e^{\kappa t}, \] (4.15)
and substitute this in eqn. (4.14). This gives four independent solutions with corresponding real constants \( \phi_0^0 \), where \( j \) runs from 1 to 4. The field solutions read,
\[ \phi_\pm = \sum_{i=1}^{4} \phi_0^\pm e^{\kappa_i t}, \quad \kappa_{\pm\pm} = -\frac{3}{2} H \left( 1 \pm \sqrt{1 - \beta \mp \gamma} \right), \] (4.16)
\[ \beta = \frac{8R \omega \bar{\mu}_r}{9H^2(\rho^2 - \omega^2)}, \quad \gamma = \frac{8R}{9H^2(\rho^2 - \omega^2)} \sqrt{(\rho^2 - \omega^2) \bar{\mu}_i^2 + \rho^2 \bar{\mu}_r^2}. \]

As can easily be verified, only \( \kappa_- \) and \( \kappa_+ \) have the possibility to become positive and hence can give growing solutions. For certain values in parameter space \( \kappa_- \) can become complex, though. In order to have an enhancement in the current, we need two independent solutions, since the contribution of solutions of \( \phi_\pm \) with equal time behavior is zero. The leading solution comes therefore from the interference of \( \kappa_+ \) and \( \kappa_- \). Neglecting the other two decaying terms, we adopt the solution,
\[ \phi_\pm = Re \left[ \phi_1^0 e^{\kappa_- t} + \phi_2^0 e^{\kappa_+ t} \right], \] (4.17)
where we took the real part since $\phi_{\pm}$ are by definition real. Substituting this in definition (3.22) of the current we arrive at,

$$J_\phi = (\phi_1^0 \phi_2^0 - \phi_2^0 \phi_1^0) \text{Re} \left[ (\kappa_- - \kappa_+) e^{(\kappa_- + \kappa_+)t} \right].$$

(4.18)

For the current to grow we clearly need $\kappa_- + \kappa_+$ to be positive. As the reader may verify, there are two branches of solutions depending on the parameters for which the current will grow. First, $\kappa_-$ can be real but slowly decaying if $\bar{\mu}_r$ takes negative values and hence $\kappa_-$ is still growing. Second, $\kappa_+$ can be complex if $\kappa_-$ grows fast enough to compensate for the negative real part of $\kappa_+$. This can be the case for both positive and negative values of $\bar{\mu}_r$. We will only address the solutions for negative $\bar{\mu}_r$ since parameter space is much larger in this case. The solutions with corresponding conditions are given,

$$J_\phi = J_0^1 e^{(-3 + \alpha_+ + \alpha_-)Ht} \quad (\kappa_- + \kappa_+ > \gamma^2),$$

(4.19a)

$$J_\phi = J_0^2 \left[ \cos(i\alpha_- Ht) + \frac{i\alpha_-}{\alpha_+} \sin(i\alpha_- Ht) \right] e^{(-3 + \alpha_+)Ht} \quad (1 - \beta < \gamma).$$

(4.19b)

Note that $\beta$ is now negative (see eqn. (4.16)) and under the condition in the second case $\alpha_-$ becomes imaginary. The parameters are given by,

$$\alpha_- = \frac{3}{2} \sqrt{1 - \beta - \gamma} \quad \alpha_+ = \frac{3}{2} \sqrt{1 - \beta + \gamma}$$

(4.20)

$$J_0^1 = H(\alpha_+ - \alpha_-)(\phi_1^0 \phi_2^0 - \phi_2^0 \phi_1^0)$$

$$J_0^2 = H\alpha_+(\phi_1^0 \phi_2^0 - \phi_2^0 \phi_1^0)$$

---

Figure 4.1: Oscillating solution corresponding to eqn. (4.19b). The dashed line indicates the end of inflation.
4.3. DOUBLE KINETIC TERM

Figure 4.2: Large timescale oscillation according to condition (4.22).

The initial currents $J^j_0 = 0$ (j=1,2) correspond to some initial charge density that will be enhanced by this mechanism. We will specify this at the end of this section. When the above conditions are satisfied, eqn. (4.18) will grow in time, provided that the combination of prefactors do not cancel such that $J^j_0 = 0$. It turns out this is not the case. Fixing $\phi^0_{1+}$ and $\phi^0_{2+}$ the prefactor corresponds to,

$$\phi^0_{1+} \phi^0_{2+} - \phi^0_{2+} \phi^0_{1-} = \frac{\phi^0_{1+} \phi^0_{2+}}{\alpha_-} \frac{9(\rho + \omega)H^3}{4\bar{\mu}_i R} \sqrt{(\rho^2 - \omega^2)\bar{\mu}_i^2 + \rho^2 \bar{\mu}_i^2}. \quad (4.21)$$

The solutions with an oscillatory behavior will have a growing envelope, but acquire positive and negative signs due to the oscillation. The growth of the current is shown in a qualitative manner in fig. 4.1. The moment inflation ends (dashed line), the mechanism is turned off and the universe remains in an asymmetric state. If the oscillation takes place on the same time scale as the exponential growth, as displayed in the figure, the creation of baryon number is dependent on the time inflation stops. We can therefore constrain the parameter space of these solutions with one more condition,

$$\alpha_+ \ll 1 \quad \rightarrow \quad 1 - \beta \approx \gamma, \quad (4.22)$$

such that inflation has ended before the oscillating prefactors can become negative. This situation is shown in fig. 4.2. Some finetuning is required to get this result, but the parameter space for which reasonable solutions are obtained in either this sector or the non oscillating sector is quite large. The solutions corresponding to eqn. (4.19a) are shown in fig. 4.3. In this case unambiguous growth takes place as long as the condition (4.19a) for the parameter space is satisfied. It is very remarkable that this mechanism can
induce exponential growth of the density while the condensate is stretched exponentially at the same time.

What we have shown here is that in the generalized Brans-Dicke theory just introduced it is possible to have a growing classical current associated with the charge of the complex Brans-Dicke scalar. This means that a charge density at the beginning of inflation can increase in density during the exponential expansion and grow to very large values at the end of inflation. Such initial charge density will be produced by quantum fluctuations. Very tiny charge fluctuations on the quantum scale will be stretched to super Hubble scales due to the expansion of the universe. Once larger than the Hubble scale, causal contact is broken and the subsequential evolution will be classical. Conventionally it is believed that such charge or particle densities get diluted by the enormous expansion of the universe during inflation. We have shown here a counterexample. The fluctuation will inflate to cosmological scales, but at the same time the density of the condensate will increase by potentially an enormous factor. This model contains solutions in parameter space for which the final baryon to entropy ratio is easily reached. Some more attention to the fate of this condensate is given at the end of this chapter.

### 4.3.1 Powerlaw expansion

Till now we made use of the assumption that $R$ is only slowly varying in time and used the limit of exponential inflation. It is interesting to investigate the situation with powerlaw inflation however, since this is the late time behavior of the inflationary epoch in a Brans-Dicke universe. We will make
4.3. DOUBLE KINETIC TERM

the following ansatz for the fields and the scale factor $a(t)$,

$$
\phi_\pm = \phi_\pm^0 (1 + \frac{t}{b})^n, \quad a(t) = a_0 (1 + \frac{t}{b})^m
$$

To solve for the exponents $m$ and $n$ we will need the Friedman equation besides the equations of motion of the fields. For convenience we rescale $\phi_\pm$ in eqn. (4.13) in the following way,

$$
\phi_\pm \rightarrow \sqrt{2(\rho \pm \omega)} \phi_\pm, \quad \xi_\pm = \frac{\mu_r}{(\rho \pm \omega)}
$$

We take, as done in section 1.5.1 in extended inflation, the trace of the stress-energy tensor equal $\rho_v - 3p$ and adopt the equation of state $p = -\rho_v$. Here $\rho_v$ denotes the energy density of the vacuum in the trapped phase. The equations to be solved are then the Friedman equation for the scale factor and the equation of motion for the rescaled fields $\phi_\pm$,

$$
H^2 = \frac{1}{3(\xi_-\phi_-^2 - \xi_+\phi_+^2)} \sum_\pm \left[ 8\pi \rho_v + \frac{1}{2} \dot{\phi}_\pm^2 + 12H \left( \pm \xi_+ \dot{\phi}_+ \phi_- - \frac{\mu_i}{\sqrt{\rho^2 - \omega^2}} (\dot{\phi}_+ \phi_-) \right) \right]
$$

$$
\ddot{\phi}_\pm + 3H \dot{\phi}_\pm \pm 2\xi_\pm \dot{\phi}_\pm R - \frac{2\mu_i}{\sqrt{\rho^2 - \omega^2}} \phi_\pm R = 0.
$$

Note that now $H \propto t^{-1}$ and $R \propto t^{-2}$. Neglecting a small constant correction of order $O(m^{-1})$ the equality $R = 12H^2$ still holds. In the original Brans-Dicke theory the scalar is sourced by the trace of the stress-energy tensor. In this case, however, we prefer the field to be dependent on the curvature. From the first equation it is easily seen that if the energy density $\rho_v$ dominates, $n$ has to be equal to unity. This is not necessarily the case, however, since the new introduced fields will grow during inflation as shown. In the next section we will even associate one of the fields with the inflaton, making it the leading contribution to the energy content of the universe. We therefore would like to solve the coupled equations in the case in which the constant term can be neglected. Because of the complexity of the problem we have done a numerical analysis. The presence of the CP violating cross term in the equation of motion results in a shift of the field solutions slightly up and down. Since $n = -1$ is also a solution of the equation of motion, we can write $\phi_\pm$ obeying both differential equations in eqn. (4.25) as,

$$
\phi_\pm = \phi_\pm^0 t^{-1-\epsilon} + \phi_\pm^1 t^{1+\epsilon} + \phi_\pm^2 t^{-1+\epsilon} + \phi_\pm^3 t^{1+\epsilon},
$$

where $\epsilon \ll 1$ denotes the small correction due to the presence of the CP violating term. We now have two fields with two growing terms, both at a different rate. This is what we needed to get a possible enhancement from
CHAPTER 4. BARYOGENESIS IN BRANS-DICKE THEORY

the definition of the current in eqn. (3.22). Rather than using this solution we can also use the result derived in eqn. (3.18), with the potential under consideration,

\[ \frac{d}{dt}(a^3 J_\phi) = \frac{a^3}{2} \left[ -\left( \xi_+ \phi_+^2 + \xi_- \phi_-^2 \right) R + \frac{2\bar{\mu}_i}{\sqrt{\rho^2 - \omega^2}} (\phi_-^2 - \phi_+^2) R \right]. \] (4.27)

Since we have two almost linear growing solutions for both fields and \( R \) is decaying as one over the time squared, the part between the square brackets approaches a constant. This means that the current will grow linear in time as well. We have derived this result in a rather qualitative way, but the notion of interest is that anything created in a previous phase will not dilute during the phase of powerlaw inflation. On the contrary, it will still grow till inflation has completely ended, although at a much slower pace. The exponent \( m \) can and will depend on all the coupling parameters of the theory, \( \rho, \omega, \bar{\mu}_r \) and \( \bar{\mu}_i \), although its precise value is not really of importance to this discussion. This is in perfect analogy to the derivation of the solutions found for the BD scalar alone in section 1.5.1. The general picture from this and the previous section is that \( \phi \) grows by subtracting energy of the contents of the universe through the curvature to which it is coupled. During early times inflation is exponential and so will be the growth of the CP odd current due to the CP violating curvature coupling. As time goes by inflation will more and more behave powerlaw with a high power \( m \) and the production of the charged current will flatten. It will not decay, however, but remain growing with a linear behavior. Once inflation has ended, the current can decay through a baryon number conserving coupling to ordinary matter.

4.3.2 Incorporation in extended inflation

One of the interesting aspects of this model is the way it can be implemented in an extended inflation type of mechanism. When comparing the rescaled lagrangian in eqn. (4.24) with the lagrangian of curvature coupled extended inflation, eqn. (1.37) from section 1.5.1, the similarity is obvious. By setting \( \bar{\mu}_r \) to either a positive or negative value we can choose whether \( \phi_+ \) or \( \phi_- \) is Brans-Dicke type, i.e. has an opposite sign in front of the quadratic term compared to the kinetic term. With an appropriate potential, the other field should be the inflaton. In analogy to the extended inflation model of Laycock and Liddle [19], one could suggest,

\[ V(\phi) = \lambda(|\phi|^2 - |\phi_0|^2)^2 = \sum_{\pm} \Lambda(\phi_\pm^2 - |\phi_0|^2)^2 + 2\lambda \phi_+^2 \phi_-^2. \] (4.28)

The main problem is that in this case also a potential for the BD scalar is added. If present, such a potential should be very weak in order not to disturb its dynamical behavior. The weakness is naturally solved when
4.3. DOUBLE KINETIC TERM

considering \( \phi_- \) to be the BD scalar and \( \phi_+ \) the inflaton, hence \( \mu_r \) is to be taken negative. The coefficients \( \rho \) and \( \omega \) are by experiment bounded to large values but nothing protects them from being of roughly the same order. Therefore \( \xi_- \) can be much smaller than \( \xi_+ \). Making the rescaling of eqn. (4.24) the potential reads,

\[
V(\phi) = \frac{\lambda}{(\rho + \omega)}(\phi_+^2 - |\phi_0|^2)^2 + \frac{\lambda}{(\rho - \omega)}(\phi_-^2 - |\phi_0|^2)^2 + \frac{2\lambda}{(\rho^2 - \omega^2)}\phi_+^2 \phi_-^2.
\]

As can be seen \( \phi_+ \) has indeed the steepest potential, making it most suitable for the inflaton. The bilinear mixing term of eqn. (4.13) is not present in the original model either. It corresponds to a tilting of the potential in the clockwise direction and can give a direction to the symmetry breaking avoiding creation of topological defects like domain walls. If the term is too big, it will mix the \( \phi_\pm \) fields in their roles as BD scalar and inflaton. The evolution will then be as follows. During the earliest stage the curvature coupled term of the inflaton is large enough to keep it trapped in a metastable minimum causing the universe to inflate. In combination with the Brans-Dicke scalar the two fields produce a baryon number fast enough to overcome the inflationary expansion. As time evolves, the curvature will fall off as \( 1/t^2 \) according to Brans-Dicke inflation and the curvature coupling of the inflaton decays. This causes the potential to develop two minima displaced from the origin and the inflaton will slowly roll into it. The Brans-Dicke field started in a second order potential but since this one is much weaker, it does not feel its influence very strongly. When the curvature drops, the potential flattens out even more since the negative curvature coupling gets weaker. Finally the inflaton reaches the true vacuum and carries together with the Brans-Dicke scalar the baryon number needed to produce ordinary matter through a decay to ordinary matter. We will not address this process in this thesis in detail. Concerning the spectrum of density perturbations this model is not much different than the model of Laycock and Liddle, treating this problem in the reference [19]. Other problems facing extended inflation are similar in this theory or might even be less severe. Since a weak potential for the BD scalar is added in a natural way, the scalar can settle down in one of the minima at late times. In this way, the value of the coupling is not restricted anymore and in that sense much more freedom is added to the model. The problem then arising is that the vacuum expectation value of the BD needs to be the gravitational constant. Some finetuning of the parameters will be needed to achieve this. The last feature of this model that might be interesting to investigate further is the last bi-quadratic term of the potential in eqn. (4.29). Such a term arises in the preheating scenario pointed out in section 2.3.2. When inflation has ended, coherent oscillations of the inflaton may give rise to a high production of BD particles. We should also mention that the physics during the rolling and oscillatory phase is still clouded since
CHAPTER 4. BARYOGENESIS IN BRANS-DICKE THEORY

the dynamics of the fields are much harder to calculate. Further analysis of these features is beyond the scope of this thesis and will be postponed till later investigation.

4.4 Higher order terms

We will now focus on the single kinetic term of eqn. (4.6), which is in need of the introduction of higher order terms to include CP violation. In this case the total action becomes,

\[ S = \frac{1}{16\pi} \int d^4 x (-g)^{1/2} \left[ I(\phi, \phi^*) R - \omega(\partial_\mu \phi \partial^*_\lambda \phi) g^{\mu\lambda} \right], \]  

(4.30)

where the lagrangian of ordinary matter and radiation is omitted since it will not play any role in the discussion. The complex phase in front of the quadratic terms can be rotated away as shown and we therefore have to look at the terms of order three and four. In order to be able to solve the equation of motion from the total action (4.30) we will make some approximations.

First of all we take the expectation value of the interaction terms so we can use the mean field approximation. Splitting the field again in a real and imaginary part such that \( \phi = \phi_r + i\phi_i \) this means,

\[ \langle \phi_r^2 \rangle = \langle \phi_i^2 \rangle = i\Delta(x, x). \]  

(4.31)

Here \( i\Delta(x, x) \) denotes the propagator of the field taken at coincidence and \( \langle \cdot \rangle \) corresponds to taking the vacuum expectation value. Furthermore the expectation value of the product of the real and the imaginary field is taken to be zero. This means no, or at least very weak, mixing between \( \phi_r \) and \( \phi_i \) at the quadratic order. The cubic and quartic terms containing only \( \phi \) or only \( \phi^* \) will therefore vanish, since we have,

\[ \langle \phi^2 \rangle = \langle \phi_r^2 \rangle - \langle \phi_i^2 \rangle + 2i \langle \phi_r \phi_i \rangle = \phi_0^2, \]  

(4.32)

where \( \phi_0 = \phi_0(t) \) denotes the classical (homogeneous) condensate. On the other hand, the expectation value,

\[ \langle \phi\phi^* \rangle = \langle \phi_r^2 \rangle + \langle \phi_i^2 \rangle = |\phi_0|^2 + 2i\Delta(x, x). \]  

(4.33)

Using the initial field \( \phi_0 \) would give us an Affleck-Dine type of mechanism, in this case slightly different since coupled to gravity. Although this is as well an interesting case to study, we will set it in our approximation to zero since for now we are interested in the contribution of the coincidence propagator. The non-vanishing CP violating terms of all possible cubic and quartic terms are then given by,

\[ \tilde{I}(\phi, \phi^*) = \frac{\sigma}{3!} \langle \phi\phi^* \rangle \phi + \frac{\lambda}{4!} \langle \phi\phi^* \rangle \phi^2 + c.c. \]  

(4.34)
4.4. HIGHER ORDER TERMS

To simplify notation we here introduced from eqn. (4.7) the coefficients $\sigma = 2\sigma_2$ and $\lambda = 3\lambda_2$. We should note that the mean field approximation as used here is often a good approximation for a rough estimate. To get a more detailed and accurate result a more thorough discussion including a full perturbative loop analysis is needed. This is beyond the scope of this thesis however. The remainder of this chapter will be dedicated to calculating this coincidence propagator and solving the resulting equation of motion.

4.4.1 Cubic term

Besides the already considered quadratic term we first include the extra cubic terms and have the following equation of motion,

$$\Box \phi + \frac{R}{\omega} \phi^* + \frac{\sigma R}{3! \omega} \phi^2 + \frac{2 \sigma^* R}{3! \omega} \phi \phi^* = 0.$$  (4.35)

To solve this non-linear differential equation we will use the mean field approximation as described above. Expressing the result in the $\phi_\pm$ representation with $\phi = \phi_+ + i \phi_-$ this yields,

$$\Box \phi_\pm \pm \frac{R}{\omega} \phi_\pm + \frac{\sigma_\pm R}{\omega} \langle \phi_\pm \phi_\pm \rangle = 0,$$  (4.36)

where $\langle \phi_\pm \phi_\pm \rangle = i \Delta(x,x)$ is the coincidence propagator. To calculate $i \Delta(x,x)$ we introduce $z$ as a function of the geodesic distance $\bar{y}(x,x')$, defined by,

$$z = 1 - \frac{1}{4} \bar{y}_{bb'}(x,x') \quad \text{and} \quad \bar{y}_{bb'}(x,x') = a(\eta) a(\eta') H^2 \Delta x_{bb'}^2$$  (4.37)

$$\Delta x_{++}^2 = ||\vec{x} - \vec{x}'||^2 - (|\eta - \eta'| - i\epsilon)^2$$

$$\Delta x_{+-}^2 = ||\vec{x} - \vec{x}'||^2 - (|\eta - \eta' + i\epsilon|^2.$$  

Here conformal time $\eta$ is used, defined by $d\eta = dt$ and the scale factor in conformal time, $a(\eta) = (-H\eta)^{-1}$. When putting this definitions into the equation of motion neglecting the interaction term we can calculate propagators with different signs $bb'$ of $\bar{y}_{bb'}$. The Feynman propagator $i \Delta_F$ will then be given by,

$$i \Delta_F = i \Delta(\bar{y}_{++}) + i \Delta(\bar{y}_{+-}).$$  (4.38)

To regulate singularities we will consider the case in $D$ dimensions first in accordance with the dimensional regularisation procedure (’t Hooft). Later on we can isolate the singularities and take the limit of $D \to 4$ again. After substituting the definition of the geodesic distance into our lagrangian the field can be rotated to decouple the real and imaginary part. We obtain [54],

$$z(1-z) \frac{d^2}{dz^2} i \Delta_\pm + D(1-2z) \frac{d}{dz} i \Delta_\pm \pm \frac{R}{\omega} i \Delta_\pm = 0.$$  (4.39)
Here $i\Delta_{\pm}$ is a vector of the plus- and minus scalar component of the Feynman propagator. This differential equation can be solved by hyper-geometric functions $\binom{a, b; c; z}{D}$. The solution reads,

$$i\Delta_{\pm} = \frac{\Gamma\left(\frac{1}{2}(D-1) + \nu_{\pm}\right) \Gamma\left(\frac{1}{2}(D-1) - \nu_{\pm}\right)}{4\pi^{D/2}\Gamma\left(\frac{D}{2}\right)} H^{D-2} 2F1\left(\frac{1}{2}(D-1) + \nu_{\pm}, \frac{1}{2}(D-1) - \nu_{\pm}, \frac{D}{2}, z\right)$$

$$\nu_{\pm} = \frac{1}{2} \sqrt{(D-1)^2 + \frac{4R}{\omega H^2}}. \tag{4.40}$$

To make a workable solution of this expression we write the hyper-geometric function in a power series and substitute the new variable,

$$s_{\pm} = \frac{D-1}{2} - \nu_{\pm}, \tag{4.41}$$

which is small when $\omega$ is large. We can now expand around $s_{\pm} = 0$ and $\epsilon = D - 4 = 0 \ [54]$. To calculate the coincidence propagator we take $\bar{y} \to 0$ in the geodesic coordinate system as well. The final expression still contains isolated singularities, which can be removed by adding counter terms. The final part of the propagator is of interest to us,

$$i\Delta_{\pm} \bigg|_{\bar{y} \to 0} = \frac{3H^2}{8\pi^2} \frac{1}{s_{\pm}}. \tag{4.42}$$

Since $s_{\pm}$ is constant, this propagator is constant and the final equation of motion of the case under consideration becomes,

$$\square \tilde{\phi}_{\pm} + \frac{1}{\omega} \tilde{\phi}_{\pm} R = 0 \tag{4.43}$$

Where $\tilde{\phi}_{\pm}$ is a shifted field given by,

$$\tilde{\phi}_{\pm} = \phi_{\pm} \mp \frac{3H^2}{8\pi^2} s_{\pm} \sigma_{\pm}. \tag{4.44}$$

The extra term in the Lagrangian induced only this shift of the field. The equation of motion of the shifted field is again of the same form as the case described in the previous section with no CP violation. Hence there will be no contribution to a CP violating current at least when using the mean field approximation.

### 4.4.2 Quartic term

In the equation of motion including the quartic terms we can use the procedure with the same propagator and mean field approximation. From the equation of motion,

$$\square \phi + \frac{R}{\omega} \mu^* \phi^* + \frac{\lambda R}{4!\omega} \phi^3 + \frac{3\lambda^* R}{4!\omega} \phi^2 \phi^* = 0, \tag{4.45}$$

56 Baryogenesis in Brans-Dicke theory
the approximated equation, in terms of vector $\phi_i^r$ of the real and imaginary part of the field, becomes,

$$\Box \phi_i^r + \frac{R}{\omega} \Omega \phi_i^r + \frac{3R}{4!\omega} (\phi \phi^*) \Lambda \phi_i^r = 0 .$$  \hspace{1cm} (4.46)

The matrix $\Lambda$, indicating the presence of mixing between the real and imaginary part of the field, is given by,

$$\Lambda = \begin{pmatrix} \lambda_r & -\lambda_i \\ -\lambda_i & -\lambda_r \end{pmatrix}, \hspace{1cm} \Omega = \begin{pmatrix} \mu_r & -\mu_i \\ -\mu_i & -\mu_r \end{pmatrix}. \hspace{1cm} (4.47)$$

This equation can be diagonalized by a proper rotation of the fields. Let $M(\theta)$ be the matrix of a standard $O(2)$ rotation over an angle $\theta$, we then have,

$$\Box \phi_{\pm} + \frac{R}{\omega} \sqrt{a^2 + b^2} \phi_{\pm} = 0 \hspace{1cm} (4.48)$$

with $\phi_{\pm} = M(\theta) \phi_i^r$ and $\theta = \frac{1}{2} \arctan \frac{a}{b}$

$$a = \mu_i + \frac{3H^2}{64\pi^2s} \lambda_i \hspace{1cm} \text{and} \hspace{1cm} b = \mu_r + \frac{3H^2}{64\pi^2s} \lambda_r .$$

The resulting equation resembles eqn. (4.43) although in this case not a field shift but a shift in effective mass is the result of the added terms. This mass shift still contains a CP violating phase, however, and this case can therefore have a charge production similar to the case studied in section 4.3. Since the two cases are comparable, we will not further analyze this particular model.

We can conclude here that in the case of the scalar being massive minimally coupled, only a quartic term as introduced above will add any contribution to a CP-violating current. In the cubic case the addition will only induce a shift in the field. We can however also assume that at the time baryogenesis is coming into play the field can be treated as a massless minimally coupled scalar. This is justified since we demanded that the effective mass in the equation of motion be typically very small compared to the energy scale of inflation. This is necessary since if not, the scalar would behave as an ordinary massive scalar. The effective mass term, present because of the quadratic term in our original Lagrangian, becomes only important after enough e-foldings. The number of e-foldings is roughly given by the logarithm of the expansion $a$. The mass term will be too small to have an important influence during inflation. Therefore for some time the field behaves as a massless scalar for which the corresponding propagator has to be used. Since this propagator is time dependent the physical system changes significantly, as we show in the next section.
4.5 Massless minimal coupling

The propagator of a massless minimally coupled scalar in $D$ dimensions is given by the differential equation,

$$\eta^\mu_\nu \partial_\mu (a^{D-2} \partial_\nu i \Delta_A(x, x')) = i \delta^D(x - x').$$  \hspace{1cm} (4.49)

The general solution for $i \Delta_A(x, x')$ is given by the expression [55, 56],

$$i \Delta_A(x, x') = H^{D-2} \frac{(D-2)}{(4\pi)^{D/2}} \frac{\Gamma(D/2)}{\Gamma(D-1)} \left( \frac{4}{\bar{y}} \right)^{D/2-1} - \pi \cot(\frac{\pi}{2}) + \ln(aa').$$  \hspace{1cm} (4.50)

To isolate dimensional and coordinate singularities we first have to expand this result in $\epsilon = D - 4$ and $\bar{y}$, since eventually we are again interested in the coincidence propagator in which $\bar{y} \to 0$. After taking the limit $\epsilon \to 0$ we are left with,

$$i \Delta_A(x, x') |_{\epsilon \to 0} = H^2 \frac{1}{4\pi^2} \left[ \frac{1}{\bar{y}} - \frac{1}{2} \ln(H^2 \Delta x^2) + \beta \right].$$  \hspace{1cm} (4.51)

where $\beta$ contains some constants and thus represents a shift of the condensate $\phi_0$ in the mean field approximation of eqn. (4.33). We will neglect this term as pointed out before. Taking the limit in $\bar{y}$ we arrive at the coincidence propagator,

$$i \Delta_A(x, x') |_{\bar{y} \to 0} = H^2 \frac{1}{4\pi^2} \ln(a) + (UV),$$  \hspace{1cm} (4.52)

where the $(UV)$ term is a diverging term in the UV regime that can be removed by adding counter terms to the lagrangian. This propagator is clearly time-dependent and when substituted in the mean field approximation we will have some nontrivial linear differential equations.

4.5.1 Cubic term

The equation of motion, including a cubic term and after the usual rotation to decouple the real and imaginary part, now reads,

$$\square \phi_\pm + R \phi_\pm + \frac{\sigma_\pm H^2 R}{8\pi^2} \ln(a) = 0.$$  \hspace{1cm} (4.53)
We will first investigate its behavior in the $k \to 0$ limit. We use that during inflation $R = 12H^2$ and the scale factor $a = e^{Ht}$. Writing out the definition of the box in Euclidean time with no space dependence, the differential equation reads,

$$
\ddot{\phi}_\pm + 3H\dot{\phi}_\pm \pm \frac{12H^2}{\omega} \phi_\pm + \frac{3\sigma_\pm H^5}{\pi^2\omega} t = 0 .
$$

(4.54)

The solution to this equation is given by,

$$
\phi_\pm = \frac{H^2\sigma_\pm}{48\pi^2}(\omega \mp Ht) + c_{1\pm} e^{-\frac{3}{2}Ht} e^{H\nu_\mp t} + c_{2\pm} e^{-\frac{3}{2}Ht} e^{-H\nu_\pm t} .
$$

(4.55)

Here the exponent is $\nu_\pm = \sqrt{\frac{9}{4} \pm \frac{12}{\omega}}$ and the $c_{i\pm}$ are just constants of integration. Now we have a wave function, we can examine whether in this particular case the field gives rise to a net current production. When substituting this result in the definition of the classical current defined in eqn. (3.22), we obtain,

$$
J_{\phi,cl} = \sum_\pm \left[ \frac{\omega H^3\sigma_\pm c_{1\pm}}{48\pi^2} e^{-\frac{3}{2}Ht} e^{H\nu_\mp t} + \frac{H^3\sigma_\pm c_{1\pm}}{48\pi^2}(\omega \pm Ht)(\frac{3}{2} - \nu_\pm) e^{-\frac{3}{2}Ht} e^{H\nu_\pm t} \right] + \frac{2\omega^2 H^5\sigma_+\sigma_-}{48^2\pi^2} + f(t) .
$$

(4.56)

The CP even terms and all decaying terms are of no importance and collected in the function $f(t)$. Eqn. (4.56) clearly contains terms growing in time. Indeed for any value of $\omega$ it is easily seen the exponent $\nu_-$ according to eqn. (4.40) will obey,

$$
\nu_- > \frac{3}{2} .
$$

(4.57)

This is the case in the first two terms when choosing the lower sign. As can be seen, picking the lower sign of the corresponding coefficients multiplies the growing exponent with the CP even $\sigma_+$ and CP odd $c_{1-}$. It seems CP violation enhances an initial classical charge asymmetry, although not in an explicit way through the coupling constant but rather by a less explicit initial condition. The CP odd part of $\sigma$ is multiplied by the exponent $\nu_+ - \frac{3}{2}$, which is negative and therefore decaying. For certain values of $\omega$ the values of $\nu_+$ will become imaginary, leading to oscillatory behavior. However, these terms will be strongly damped by the inflationary expansion as well. Also because of the present lower boundary for $\omega$ at 40000 this would require a mechanism for $\omega$ to change its value in time. The last term in eqn. (4.56) is constant but contains explicit CP violating through $\sigma_-$ and not by initial conditions. It means that a patch containing an asymmetry of charge will grow by inflation without changing the charge density. In a way charge is continually generated to keep up with the expansion.
We will now investigate further the properties of the full space dependent equation of motion including inhomogeneities,

\[ \ddot{\phi}_\pm + 3H \dot{\phi}_\pm - \left( \frac{\nabla}{a} \right)^2 \phi_\pm \pm \frac{12H^2}{\omega} \phi_\pm + \frac{3\sigma_\pm H^5}{\pi^2 \omega} t = 0. \] (4.58)

This is now a differential equation of the quantum mechanical operator fields \( \hat{\phi}_\pm \). We can perform a time dependent shift of the field in order to get rid of the problematic last term. The appropriate shift is,

\[ \delta \phi_\pm = \frac{H^2 \sigma_\pm}{4\pi^2} \left( \pm Ht + \frac{1}{4} \omega \right). \] (4.59)

The equation of motion of the shifted field \( \hat{\phi}_{s\pm} = \hat{\phi}_\pm + \delta \hat{\phi}_\pm \), with \( \delta \hat{\phi}_\pm \) the commuting part stated in eqn. (4.59), has then the form,

\[ \ddot{\phi}_{s\pm} + 3H \dot{\phi}_{s\pm} - \left( \frac{\nabla}{a} \right)^2 \phi_{s\pm} \pm \frac{12H^2}{\omega} \phi_{s\pm} = 0. \] (4.60)

The operator fields \( \hat{\phi}_{s\pm} \) can be expanded in mode functions \( \varphi_{\pm}(\vec{k}, \eta) \) and hence we write,

\[ \hat{\phi}_{s\pm}(\vec{x}, \eta) = \frac{1}{a(\eta)} \int \frac{d^3\vec{k}}{(2\pi)^3} \left[ e^{i\vec{k}\vec{x}} \varphi_{\pm}(\vec{k}, \eta) \hat{a}_\pm(\vec{k}) + h.c. \right]. \] (4.61)

Here \( \hat{a}_\pm(\vec{k}) \) denotes the annihilation operator of a particle of the \( \pm \) field with momentum \( \vec{k} \), obeying the commutation relations,

\[ [\hat{a}_b(\vec{k}), \hat{a}_b^\dagger(\vec{k}')] = (2\pi)^3 \delta^3(\vec{k} - \vec{k}') \delta_{bb'} . \] (4.62)

Here \( b \) denotes the + or - field. In order to calculate the mode functions it is more convenient to use conformal time \( \eta \) as defined before. In the differential equation we therefore have to transform the time derivatives to conformal time derivatives, implying \( \dot{\phi} \rightarrow \frac{1}{a} \phi' \). The derivative with respect to conformal time is denoted by the prime. Since we would like to know the equation of motion of the mode functions \( \varphi \), we also make the substitution \( \phi \rightarrow \varphi/a \) and replace \( -i\nabla \varphi \rightarrow k\varphi \). Eqn. (4.60) then becomes,

\[ \varphi''_\pm + \left[ k^2 - \frac{a''}{a} \pm \frac{a^2 R}{\omega} \right] \varphi_\pm = 0. \] (4.63)

During inflation \( R = 12H^2 \) and \( a = (-H\eta)^{-1} \) and this becomes,

\[ \varphi''_\pm + \left[ k^2 - \frac{2}{\eta^2} \pm \frac{12}{\omega} \right] \varphi_\pm = 0. \] (4.64)
4.5. MASSLESS MINIMAL COUPLING

The solution to this differential equation is given by Hankle functions,

\[ \varphi_{\pm}(k, \eta) = \frac{1}{2} \sqrt{-\pi \eta} H_{\nu_{\pm}}^{(1)}(-k\eta) \quad \text{and} \quad \nu_{\pm} = \frac{3}{2} \sqrt{1 \pm \frac{16}{3\omega}}. \]  

(4.65)

In this quantum mechanical case the current can be written,

\[ \langle 0 | J_\phi | 0 \rangle = J_{\phi, cl} - i \frac{1}{2} \sum_{\pm} \int \frac{d^3k}{(2\pi)^3} \frac{1}{a^3} W[\varphi_{\pm}, \varphi_{\pm}^*]. \]  

(4.66)

Here is \( |0\) the Bunch-Davies vacuum and \( W[\psi, \psi^*] = \dot{\psi}\psi^* - \psi\dot{\psi}^* \) denotes the Wronskian of the field. The latter one is known to be conserved and equal to \( i \) for the Hankel functions we just derived. In physical time this yields \( i/a \). This means the current due to the shifted field \( \phi_{s\pm} \) will be conserved and looks like,

\[ \langle J_\phi \rangle = J_{\phi, cl} + \sum_{\pm} \int \frac{d^3k}{(2\pi)^3} \frac{1}{a^3} \frac{1}{2}. \]  

(4.67)

This is just the classical current derived in eqn. (4.66) due to the shift of eqn. (4.59) plus the quantum mechanical number density of the particles in vacuum. The shift can be identified as a classical condensate, while the solution of the shifted field give quantum mechanical modes on top of it. Since the shift corresponds to the homogeneous solution of the classical equation of motion given in eqn. (4.54), the complete solution of the classical condensate is the same as calculated in eqn. (4.55). The current due to the quantum fluctuations given by the Hankel functions give no growing solutions. The classical condensate, however, is added to each mode and does grow. Imposing initial conditions like,

\[ \phi_{cl\pm}(0) = 0 \quad \text{and} \quad \dot{\phi}_{cl\pm}(0) = 0, \]  

(4.68)

destroys the classical solution and leaves only the decaying quantum fluctuations. This means we have the same situation as at the end of section 4.3. When initial fluctuations become super Hubble, they will be enhanced by this mechanism.

4.5.2 Quartic term

In this section we consider the lagrangian including the quartic terms from eqn. (4.34). After transformations we calculate the equation of motion to be,

\[ \square \phi_{\pm} + \frac{R}{\omega} \phi_{\pm} + \frac{H^2 R}{8\pi^2 \omega} \ln(a) \Lambda \phi_{\pm} = 0. \]  

(4.69)

Here denotes \( \Lambda \) again the mixing matrix given in eqn. (4.47). This differential equation can not be solved analytically and therefore we have to
use perturbative techniques to solve it. As unperturbed field we take the solution of the differential equation,

\[ \Box \phi^{(0)}_\pm \pm \frac{R}{\omega} \phi^{(0)}_\pm = 0. \] (4.70)

The solution is given by (see also eqn. (4.55)),

\[ \phi^{(0)}_\pm(t) = c_1^\pm e^{-\frac{3}{2}Ht} e^{H\nu_\pm t} + c_2^\pm e^{-\frac{3}{2}Ht} e^{-H\nu_\pm t}. \] (4.71)

Here \( c_1^\pm \) and \( c_2^\pm \) are constants of integration and the Ricci scalar \( R \) is again taken to be \( 12H^2 \). The first order perturbation is then written,

\[ \phi^{(1)}_\pm(t) = \int_{-\infty}^{\infty} iG(t-t') \left[ \frac{RH_3^\pm t'}{8\pi^2\omega} \left( \pm \lambda_i \phi^{(0)}_\pm(t') - \lambda_i \phi^{(0)}_\mp(t') \right) \right] dt'. \] (4.72)

Here the definition of the matrix \( \Lambda \) in eqn. (4.47) is used and \( iG_\pm(t-t') \) denote the retarded Green functions associated with the differential equations of eqn. (4.70), given by,

\[ iG_\pm(t-t') = \Theta(t-t') \frac{1}{H\nu_\pm} \sinh(H\nu_\pm(t-t')) e^{-\frac{3}{2}H(t-t')} . \] (4.73)

Here \( \Theta(t-t') \) is the Heaviside step function. As should be clear, only the terms multiplying \( \lambda_i \) are CP violating and moreover, the decaying terms appearing in \( \phi^{(0)}_\pm \) will not be important when considering late time behavior. We therefore end up with,

\[ \phi^{(1)}_\pm(t) = \frac{RH^2\lambda_i c_1^\pm e^{-\frac{3}{2}Ht}}{16\pi^2\omega \nu_\pm} \int_{-\infty}^{t} \left( e^{-H\nu_\pm t} e^{H(\nu_+ + \nu_\mp)t'} - e^{H\nu_\pm t} e^{H(\nu_\mp - \nu_\pm)t'} \right) dt' \]

\[ = \frac{H^2\lambda_i c_1^\pm}{194\pi^2} \left[ \frac{\nu_+ \omega}{6} - Ht \right] e^{H(\nu_\pm - \frac{3}{2})t} + l(t, t_i) . \] (4.74)

Here \( l(t, t_i) \) is a function of \( t \) and some initial time \( t_i \). As the reader may verify, these terms will not give a dominant contribution to the current since they contain only exponentials of \( \nu_\pm \) instead of \( \nu_\mp \), which is needed to obtain a growing solution. Since no CP violation occurs in the leading order, the subdominant CP odd current can be written as,

\[ J_{\phi,cl} = \phi^{(0)}_+ \phi^{(1)}_+ - \phi^{(0)}_- \phi^{(1)}_- - \phi^{(0)}_+ \phi^{(1)}_- + \phi^{(0)}_- \phi^{(1)}_+ \approx \frac{H^3\lambda_i (c_1^\pm)^2}{194\pi^2} e^{H(2\nu_\mp - 3)t} . \] (4.75)

Decaying terms have been omitted in the last expression. Similar to the homogeneous case of cubic interaction terms, an exponential growth in the
4.5. MASSLESS MINIMAL COUPLING

zero mode is obtained. This time the CP violation is explicitly coming from the imaginary part of the coupling constant $\lambda$. From the calculations it becomes clear that the mixing between the plus and minus field due quartic term, visualized in the $\Lambda$ matrix, is the direct cause for the current production in eqn. (4.75).

We finally calculate a perturbative solution to the equation of motion (4.69) in a full quantum mechanical treatment including quantum fluctuations of nonzero modes. In this case it is more convenient to use the equation of motion of the complete complex field $\phi$ instead of splitting it up in the $+/-$ fields,

\[ \Box \phi + \frac{R}{\omega} \phi^* + \frac{H^2 R}{8\pi^2 \omega} \ln(a) \lambda^* \phi^* = 0 . \]  

As unperturbed field we take the solution of the massless scalar,

\[ \Box \phi^{(0)} = \frac{1}{a^2} \partial^\mu (a^2 \partial_\mu \phi^{(0)}) = 0, \]  

where the scale factor is now a function of conformal time $\eta$. The solution of the mode functions $\varphi^{(0)}(\vec{k}, \eta)$ to this equation is given by,

\[ \varphi^{(0)}(\vec{k}, \eta) = \frac{\alpha}{\sqrt{2k}} (1 - i \frac{\eta}{k}) e^{-i k \eta} + \frac{\beta}{\sqrt{2k}} (1 + i \frac{\eta}{k}) e^{i k \eta}, \]  

where the coefficients are constrained by $|\alpha| - |\beta| = 1$ and we made the substitution $\phi \rightarrow \varphi/a$. We choose the Bunch-Davies vacuum, $|\alpha| = 1$ and $|\beta| = 0$. The field can therefore be written as,

\[ \phi^{(0)}(x) = \frac{1}{a} \int d^4 x' \frac{\delta^3(\vec{k} - \vec{k}')}{(2\pi)^3} \left[ \frac{e^{i \vec{k} \vec{x}}}{\sqrt{2k}} (1 - i \frac{\eta}{k}) e^{-i k \eta} \hat{a}(\vec{k}) + \frac{e^{-i \vec{k} \vec{x}}}{\sqrt{2k}} (1 + i \frac{\eta}{k}) e^{i k \eta} \hat{b}^\dagger(\vec{k}) \right], \]  

where, since we now deal with a complex field, $\hat{a}(\vec{k})$ denotes the annihilation operator of a particle and $\hat{b}^\dagger(\vec{k})$ the creation operator of an anti-particle. The non vanishing commutation relations are,

\[ [\hat{a}(\vec{k}), \hat{a}^\dagger(\vec{k}')] = [\hat{b}(\vec{k}), \hat{b}^\dagger(\vec{k}')] = (2\pi)^3 \delta^3(\vec{k} - \vec{k}') . \]  

All others vanish. For convenience we work with conformal time $\eta$ again. The first order correction to this unperturbed field using the retarded Green function $i \Delta_{ret}(x; x')$ of the massless scalar yields,

\[ \phi^{(1)}(x) = \int d^4 x' i \Delta_{ret}(x; x') \left[ \frac{R}{\omega} + \frac{H^2 R \lambda^*}{16\pi^2 \omega} \ln(a) \right] \phi^{(0)*}(x') . \]  

The expression between the square brackets is the perturbation. Using the definitions of eqn. (4.37) we can rewrite the Feynman propagator of a massless
CHAPTER 4. BARYOGENESIS IN BRANS-DICKE THEORY

scalar as given in eqn. (4.51) to get,

\[ i\Delta_{ret}(x; x') = -\frac{iH^2}{2\pi^2} \left[ \eta\delta(u^2) + \frac{1}{2} \Theta(u^2) \right] \Theta(\eta - \eta') \]  

(4.82a)

\[ u^2 = (\eta - \eta')^2 - \|\vec{x} - \vec{x}'\|^2 . \]  

(4.82b)

We would like to solve the integral over the spatial part in the correction term first. To do this we introduce the spatial distance function

\[ r = \|\vec{x} - \vec{x}'\| \]

and transform the spatial part to spherical coordinates. The integral is then solved using,

\[ \int d^3 r g(r) \frac{e^{i\vec{k}\vec{r}}}{kr} = 4\pi \int dr r^2 g(r) \int_{-1}^{1} \frac{d\cos(\theta)}{2} e^{ikr\cos(\theta)} \]

\[ = 4\pi \int dr r g(r) \sin(kr) . \]  

(4.83)

Here \( g(r) \) contains all \( r \) dependent terms. After integrating out \( r \) and using that during inflation \( 1/\alpha = -H\eta \) we calculate the mode functions to the first order in perturbation to be,

\[ \phi^{(1)}(k, \eta) = \frac{iH^3}{\sqrt{2\pi}} \int_{\eta_0}^{\eta} d\eta' \left( \eta' - \frac{i}{k} \right) e^{-i\eta'k} \left[ \frac{R}{\omega} - \frac{H^2 R\lambda^*}{16\pi^2\omega} \ln(-H\eta') \right] \]

\[ \times \left[ (k^2\eta\eta' + 1) \sin k(\eta - \eta') - k(\eta - \eta') \cos k(\eta - \eta') \right] . \]  

(4.84)

The integration is performed from some initial time \( \eta_0 \) to some later time \( \eta \). Since we are interested in the CP violating contribution to the current, the most important terms are those multiplied by the imaginary part \( \lambda_i \) of the coupling. The leading CP violating contribution to the current is of order \( O(\omega^2) \) and comes from these terms, in combination with the unperturbed mode function \( \phi^{(0)} \). Collecting the CP violating terms in \( \phi^{(1)}_{CP} \) we can write the current,

\[ J_{CP}^{(1)} = \frac{i}{2} \int \frac{d^3k}{(2\pi)^3} \frac{1}{a^2} \left( W[\phi^{(0)}, \phi^{(1)*}_{CP}] + W[\phi^{(1)}_{CP}, \phi^{(0)*}] \right) . \]  

(4.85)

We will not write down the complete expression for the two Wronskians but rather point out what assumptions have to be made to extract the leading order of it. The integrand of eqn. (4.85) contains terms of the initial modes \( -k\eta_0 \gg 1 \) and the late time modes \( -k\eta \ll 1 \). First of all, an expansion in the late time modes can be made. Using the asymptotic expansion for the harmonic integral functions of the initial modes, we see that these terms can be neglected. Finally we can assume the initial scale factor to be unity \( a_0 = a(\eta_0) = 1 \), a freedom we have, and several terms drop out. We are left

64

Baryogenesis in Brans-Dicke theory
As can be seen, this integral has an infrared divergence and needs to be cut off. The previous made assumptions of the modes provide a natural cut off for the theory. From $-k\eta_0 = \frac{k}{H a_0} \gg 1$ we identify a minimal mode $k_{\text{min}} = H$ and from $-k\eta \ll 1$ a maximal mode $k_{\text{max}} = H a(\eta)$. The mode corresponding to $k_{\text{min}}$ is the one just crossing the horizon at initial time $\eta_0$. On the contrary, $k_{\text{max}}$ corresponds to the subhubble mode at time $\eta_0$, just becoming superhubble at time $\eta$. When evaluating the integral the (constant) leading order drops out and as a result the current scales as one over the scale factor in the highest subleading order. The oscillating term might be a consequence of the perturbative treatment used and we suspect it not to appear in an exact treatment. Since a normal scalar current scales as one over the scale factor cubed, enhancement is achieved. It is not sufficient to enhance initial charge fluctuations to galactical scales, though, and therefore not of importance in this model.

### 4.6 Fate of the condensate

In this chapter we have shown that the complex scalar in the generalized Brans-Dicke theory gives rise to a growing classical current. The field is assumed to carry a charge $Q$, which is a linear combination of both $B$ and $L$,

$$Q = q_B B + q_L L = \epsilon(B + L) + \delta(B - L) .$$

(4.87)

Here denote $\epsilon$ and $\delta$ the fractions of the charge proportional to the orthogonal currents $B + L$ and $B - L$ respectively. Eventually inflation will end and we are left with a large condensate of charge $Q$. Till now, a similar mechanism could have been constructed with the replacement of $R \to m^2$, with $m$ a large mass scale. That would mean that CP violation remains operative till even today, causing effects not seen yet in experiment. The curvature coupling, on the contrary, is turned off in the late time universe, making this type of mechanism favorable. The charged current can decay through a tree level coupling to ordinary matter. Such a coupling will conserve the charge $Q$ of the field and hence we can write,

$$J_\phi \simeq J_Q .$$

(4.88)
The universe enters the stage of preheating in which a lot of entropy is produced, since entropy scales as $s \sim T^3$ (with $T$ the temperature). During inflation the temperature experienced by a person at static coordinates is the Hawking temperature associated with the Hubble volume. This means during inflation we have $s \sim H^3$. During preheating and the subsequent phases the inflaton thermalizes and the temperature of the universe increases. The enhancement of the entropy is roughly $\sim 10^8$. Our current universe has the baryon to entropy ratio [29],

$$\frac{n_B}{s} \equiv \frac{n_b - n_{\bar{b}}}{s} \approx (6.1 \pm 0.3) \times 10^{-10}.$$ (4.89)

We therefore have to demand our model to account for baryon production such that at the end of inflation,

$$\left. \frac{J_\phi}{s} \right|_{\text{end infl}} \sim 10^{-2}.$$ (4.90)

This seems not to be a problem with the mechanism we have presented. Similar to the discussion on leptogenesis in section 2.6, sphaleron processes will distribute the charge over lepton and baryon number but will wash away the $B + L$ component of $Q$. This process is described by Harvey and Turner for instance [49] and results in the following relations between the initial and final abundances,

$$B_f = \frac{8N + 4(m + 2)}{24N + 13(m + 2)} (B - L)_i,$$ (4.91a)

$$L_f = -\frac{16N + 9(m + 2)}{24N + 13(m + 2)} (B - L)_i,$$ (4.91b)

$$(B + L)_f = -\frac{8N + 5(m + 2)}{24N + 13(m + 2)} (B - L)_i.$$ (4.91c)

Here denotes the subscript $i$ the initial abundance, $f$ the final abundance, $N$ is the number of particle species and $m$ the number of Higgs doublets. It should be clear that however the initial $B + L$ is washed away, the initial $B - L$ is processed to a final nonzero $B + L$. 

---

Baryogenesis in Brans-Dicke theory
Chapter 5

Conclusion

We have constructed a model for baryogenesis by generalizing Brans-Dicke theory to a theory with a complex scalar field which includes CP violation. We had the choice of either use a combination of kinetic terms or use the cannonical form. In the latter case CP violation only enters the theory when adding higher order terms to the lagrangian. To be able to solve the equations of motion we used the mean field approximation. Due to the high energy scale of inflation, the effective mass term only becomes important after a sufficient number of e-foldings. We can therefore safely treat the field as a massless minimally coupled scalar. The implications are that the coincidence propagator used in the mean field approximation scales as $\log(a)$. The added CP violating interaction terms then force the field to the production of baryon number during the inflationary era. We have seen that both adding cubic and quartic interaction terms in the lagrangian only enhances the classical condensate. In the first case this effect is less explicit due to initial conditions of the field. The CP odd part of the coupling only keeps a charge distribution constant while space inflates, but can not enhance the initial charge. In the latter case the coupling of the quartic interaction explicitly drives the enhancement of the current. Some small initial charge asymmetry can be enlarged by this mechanism when super Hubble and thus classically behaving. As the universe expands, fields normally get stretched with decaying amplitude. In our case however, the amplitude gets magnified not only to overcome the expansion but grow even larger in a comoving frame. This growth is only seen in the zero mode, quantum fluctuations of nonzero momentum are again not enlarged by the mechanism. In the case of adding quartic terms, quantum fluctuations are not decaying as fast as in an ordinary scalar current. This solution is still not sufficient to prevent dilution, however. Once the enhanced homogeneous mode re-enters the horizon, it will contain a large baryon number which can decay through a coupling to ordinary matter. To fortify the conclusions it might be necessary to examine the justification of using the mean field approximation.
approximation. This is beyond the scope of the thesis, however.
The model with multiple kinetic terms possesses CP violation in the quadratic terms already. The benefit of this mechanism is that CP violation occurs at tree level, hence no suppression is present and no approximations are needed. Under certain conditions for the parameter space this mechanism can produce net baryon number in an explicit way. Also when the inflationary expansion relaxes to powerlaw still linear growth is achieved. The conditions give rise to some finetuning, but further numerical analysis might show that bounds are less severe. It has also been shown that the mechanism can fit into a model of extended inflation with a minimal adaptation when considering the real and imaginary part of the field independently. The real part acts like a Brans-Dicke scalar while the imaginary part drives the inflation. The theory also gives a natural weak potential for the Brans-Dicke scalar, potentially anchoring the expectation value today and loosen the constraints on the theory. More detailed calculations will be needed to further adjust these proposals and compare it with experimental bounds. The elegance of the generalization of Brans-Dicke theory under study in this thesis is the fact that a model for baryogenesis is implemented in the theory of gravity itself. By combining CP violation and gravitation the force responsible for the long distance interaction of matter produces matter at the same time. Another interesting aspect is that the process takes place during inflation, a very important era in the evolution of the universe not much considered in combination with baryogenesis. The CP violation in the theory is also naturally turned off when the universe enters matter era, hence today’s physics is not affected by any additional effects. It would be interesting to further investigate the model in a more general Scalar-Tensor theory of gravity or exploit the analog in string theory with the dilaton.
Appendix A

Short introduction to super symmetry

The idea for super symmetry, susy in short, came from the fact that the Standard Model lagrangian still suffers from the hierarchy problem and quadratic divergences. The first problem means the enormous difference in orders between the energies of the electroweak scale and the Planck scale. The reason why this is problematic becomes clear when considering the second problem. Quadratic divergences appear when calculating the quantum corrections to the mass of scalar particles like the Higgs particle. In the infrared limit, a cut off energy scale is therefore needed, but it should be higher than the electroweak scale. It turns out problems can be solved by imposing a symmetry between bosons and fermions called super symmetry. In this symmetry every boson has a susy partner and vice versa. Since no such a symmetry is seen in nature nor in particle accelerators, it has to be broken at present day. It is suggested the cut off appearing in the calculation of the Higgs mass, signaling new physics, is the energy scale of susy breaking (about $1000\, GeV$). The particles present in the Standard Model do not have the right properties to be each others super partners, unfortunately. A new super symmetric particle, or sparticle, for every particle in the Standard Model has to be introduced. It turns out the additional loop corrections due to the sparticles almost exactly cancel the quadratic divergences to all order loop corrections except the logarithmic ones in the Higgs mass. Sparticles differ from spin but equal all other quantum numbers to their partners. The particle field and its super symmetric sparticle field are joined in a multiplet called a super field. The light fields are described by two of such multiplets (neglecting the graviton). The chiral multiplets contain a scalar boson and a Weyl fermion, general notation,

$$\Phi_i = (\phi_i, \psi_i) \,.$$  \hspace{1cm} (A.1)
APPENDIX A. SHORT INTRODUCTION TO SUPER SYMMETRY

A gauge boson has a bosino as superpartner, also a Weyl fermion, and are joined in a vector multiplet,

\[ V^a = (A^a \mu , \lambda^a) . \]  
(A.2)

To define supersymmetry, there is a need for extending the usual 4-coordinate space-time to superspace by introducing a fermionic coordinate. Such a coordinate is a Grassmann variable \( \theta \), a spin 1/2 object carrying a mass dimension and two degrees of freedom. They are defined by the following properties,

\[
\{ \theta_{i\alpha}, \theta_{j\beta} \} = 0, \quad \theta^\alpha_i \theta^\beta_i = -\frac{1}{2} \epsilon^{\alpha\beta} \theta^\alpha_i \theta^\beta_i \]  
(A.3a)

\[
\frac{d\theta_{i\alpha}}{d\theta_{j\beta}} = \delta^i_j \delta^\alpha_\beta, \quad \int d\theta_i = 0, \quad \int d\theta_{i\alpha} d\theta_{j\beta} = \delta_{ij} \delta^\alpha_\beta . \]  
(A.3b)

Here the Latin indices label the different Grassmann variables and the Greek indices are spinorial indices. Furthermore, \( \epsilon_{\alpha\beta} \) denotes the two dimensional antisymmetric Levi-Civita tensor, having \( \epsilon_{12} = 1 \). To see how the superfields are constructed, we can extend a complex scalar like the Higgs boson \( \phi(x^\mu) \) to a superfield \( \Phi(x^\mu, \theta_\alpha) \). We will require this field to have the same quantum numbers and mass dimension as the old fashioned \( \phi(x^\mu) \) and expand it in the Grassmann variable,

\[
\Phi(x^\mu, \theta_\alpha) = \varphi(x^\mu) + \theta^\alpha \chi_\alpha(x^\mu) + \theta^\alpha \theta_\alpha F(x^\mu) , \]  
(A.4)

where \( \varphi, \chi_\alpha \) and \( F \) are functions which can be identified with certain fields or field configurations, as we will see. Due to the properties of \( \theta_\alpha \) these are in fact the only non-vanishing terms in the expansion and hence an exact equality. We will not go into too much detail about this result and only clarify the basics. First it should be clear a similar expansion can be made in case of a vectorial superfield, which we will not discuss here. In the expression of eqn. (A.4) we can identify the first field with the ordinary Higgs boson \( \varphi = \phi \). In order to satisfy the requirement that the second term behaves as a scalar under transformations, \( \chi_\alpha \) can be identified with a spin 1/2 object. Hence \( \chi_\alpha = \psi_\alpha \) in the definition of eqn. (A.1) is a fermion. From the mass dimensions we see the last (scalar) field \( F \) has dimension 2 and carries two bosonic degrees of freedom. This does not correspond to any physical particle in nature and in fact one can show that indeed this term does not correspond to a propagating field. \( F \) rather generates interactions and mass terms and is therefore an auxiliary field with no dynamical physical degrees of freedom.
A.1. SUPERSYMMETRIC LAGRANGIAN

A.1 Supersymmetric Lagrangian

Going a little bit ahead one can introduce a superpotential from which these interactions are generated,

\[ W(\phi_i) = \frac{1}{2} m_{ij} \phi_i \phi_j + \lambda_{ijk} \phi_i \phi_j \phi_k . \]  

(A.5)

Its relation with the auxiliary field is shown in a moment. We can now construct a super symmetric Lagrangian which has to contain the following terms. First of all there are kinetic terms, with \( D \) denoting the covariant derivative,

\[ \lambda^a D \lambda^{a*}, \quad \psi D \psi^*, \quad -\frac{1}{4} F_{\mu\nu}^a, \quad |D_m \phi|^2 . \]  

(A.6)

Here \( F^a_{\mu\nu} \) denotes the field strength of a gauge field labelled \( a \). Second there are Yukawa couplings with gauge strength, denoting the generators of a corresponding gauge theory by \( T^a \),

\[ \sqrt{2} g^a \lambda^a \phi^* T^a \psi + c.c. , \]  

(A.7)

and mass terms and Yukawa couplings where we make use of the definition of the superpotential,

\[ \frac{1}{2} \frac{\partial^2 W}{\partial \phi_i \partial \phi_j} \psi_i \psi_j = m_{ij} \psi_i \psi_j + \frac{3}{2} \lambda_{ijk} \phi_i \psi_j \psi_k . \]  

(A.8)

Finally scalar potential terms are again generated by the superpotential and from gauge interactions,

\[ V = \sum_i \left| \frac{\partial W}{\partial \phi_i} \right|^2 + \sum_a \frac{1}{2} \left( g^a \sum_i \phi_i^* T^a \phi_i \right)^2 . \]  

(A.9)

It turns out we can write the potential in terms of two auxiliary fields \( F \) and \( D^a \)

\[ F_i = \frac{\partial W}{\partial \phi_i}, \quad D^a = g^a \sum_i \phi_i^* T^a \phi_i . \]  

(A.10)

The potential then becomes,

\[ V = |F_i|^2 + \frac{1}{2} |D^a|^2 . \]  

(A.11)

The auxiliary field \( D^a \) comes from the expansion of the vector super field similar as in eqn. (A.4). This is not straightforward to show, but we will not explain this here. It turns out these fields are exactly sufficient to generate the interactions needed for the cancellations explained above.
A.2 MSSM

All Standard Model fields can be extended to superfields in this way. As told before no present known bosons and fermions can be combined in a susy multiplet. Therefore 14 bosinos (from 12 gauge bosons and 2 complex scalars) and 45 sfermions (from 3 families each with 15 Weyl fermions) have to be added to the known particle spectrum. All these super partners should have masses ranging from 200 up to 10 TeV in order to solve the hierarchy problem explained above. Some models require even larger masses. Furthermore, the Higgs sector needs to be extended by yet another Higgs doublet and its supersymmetric Higgsino doublet. This is because in the Standard Model the Higgs field $\phi$ generates masses for the down type quarks and charged fermions, while its charge conjugate $\phi^c = i\tau_2\phi^*$ does the same for up type quarks. The superpotential of eqn. (A.5) is a function of $\phi$ only, so another unique Higgs doublet needs to be introduced to be able to give all the particles their mass. Another reason is the need for cancellation of the anomalies arising when only one Higgsino is present in the theory. This supersymmetric extension of the Standard Model is known as the Minimal Supersymmetric Standard Model, MSSM for short. Table A.1 lists all fields, their nomenclature and notation in it [57]. Other extensions exists, the most simple being the nearly Minimal Supersymmetric Standard Model (nMSSM), in which the Higgs sector is extended with yet another Higgs singlet. The reason for this was to address the $\mu$-problem, in which the coupling $\mu$ arising in the Higgs sector has to take a value incompatible with the symmetrybreaking [58]. For baryogenesis the extension had the interesting side effect that it makes the electroweak phase transition stronger. The Next-to-Minimal Supersymmetric Standard Model (NMSSM) also contains (self) interactions of the singlet, enriching the Higgs sector even more.

For now we will focus on the MSSM. To construct a realistic model, a new conserved quantum number needs to be introduced. Particles and sparticles cannot be exchanged in interactions without violation of angular momentum. It is therefore imposed that R-parity is conserved, particles having $R = +1$ and sparticles $R = -1$. This prevents sparticles also from decaying to particles, which means the lightest supersymmetric particle (LSP) must be stable. The abundance of LSP in the universe might solve the dark matter problem. The superpotential for the MSSM is given by [57],

$$W = y_{eab}\epsilon_{ij}Q^i_aE_bH^j_1 + y_{ub}\epsilon_{ij}Q^i_aU_bH^j_1 - y_{db}\epsilon_{ij}Q^i_aD_bH^j_2 + \mu\epsilon_{ij}H^i_1H^j_2.$$  
(A.12)

This is in fact a generalization of the ordinary Yukawa couplings appearing in the Standard Model, $y_{eab}$, $y_{ub}$ and $y_{db}$ being the MSSM Yukawa coupling constants. The multiplets in eqn. (A.12) represent the complete chiral superfields and include the bosons and fermions as well as their bosinos.
and sfermions. Other terms are permitted by gauge symmetries, but would violate baryon and lepton number and are prohibited by R-parity.

<table>
<thead>
<tr>
<th>Particle</th>
<th>Sparticle</th>
<th>Boson field</th>
<th>Fermion field</th>
<th>Multiplet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Higgs</td>
<td>Higgsino</td>
<td>$H_1 = \begin{pmatrix} H_0^1 \ H_1 \end{pmatrix}$</td>
<td>$\tilde{H}_1$</td>
<td>$(1, 2)_{\frac{1}{2}}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$H_2 = \begin{pmatrix} H_0^2 \ H_2 \end{pmatrix}$</td>
<td>$\tilde{H}_2$</td>
<td>$(1, 2)_{\frac{1}{2}}$</td>
</tr>
<tr>
<td>Gluon</td>
<td>Gluino</td>
<td>$g_a$</td>
<td>$\tilde{g}_a$</td>
<td>$(8, 1)_0$</td>
</tr>
<tr>
<td>W</td>
<td>Wino</td>
<td>$W_i$</td>
<td>$\tilde{W}_i$</td>
<td>$(1, 3)_0$</td>
</tr>
<tr>
<td>B</td>
<td>Bino</td>
<td>$B$</td>
<td>$B$</td>
<td>$(1, 1)_0$</td>
</tr>
<tr>
<td>Quark</td>
<td>Squark (doublet)</td>
<td>$\tilde{Q}_n$</td>
<td>$Q_n = \begin{pmatrix} u \ d \end{pmatrix}_{L_n}$</td>
<td>$(3, 2)_{\frac{1}{6}}$</td>
</tr>
<tr>
<td></td>
<td>(singlet up type)</td>
<td>$\tilde{U}_n$</td>
<td>$U_n = u^c_{L_n}$</td>
<td>$(3, 1)_{\frac{2}{3}}$</td>
</tr>
<tr>
<td></td>
<td>(singlet down type)</td>
<td>$\tilde{D}_n$</td>
<td>$D_n = d^c_{L_n}$</td>
<td>$(3, 1)_{\frac{1}{3}}$</td>
</tr>
<tr>
<td>Lepton</td>
<td>Slepton (doublet)</td>
<td>$\tilde{L}_n$</td>
<td>$L_n = \begin{pmatrix} \nu \ e^- \end{pmatrix}_{L_n}$</td>
<td>$(1, 2)_{\frac{1}{2}}$</td>
</tr>
<tr>
<td></td>
<td>(singlet)</td>
<td>$\tilde{E}_n$</td>
<td>$E_n = (e^-)^c_{L_n}$</td>
<td>$(1, 1)_{1}$</td>
</tr>
</tbody>
</table>

Table A.1: Particles and their fields in the MSSM. All bosons carry spin 1 except for the Higgs particles and all fermions carry spin 1/2. The labels $a$ denote the eight gluon fields, $i$ the three weak fields and $n$ the three fermion families. The multiplets are denoted as $(Q_c, Q_L)_{Q_{Y/2}}$ with respectively the color charge, weak isospin and hypercharge. The combination $(1, 1)_0$ transforms as a singlet under all symmetries. The graviton is not taken into consideration.

### A.3 Flat directions

A flat direction is a direction in field space along which the potential vanishes. In the ordinary Standard Model this is not possible to construct, but due to the large amount of fields, flat directions in the MSSM are numerous. That is, only if just the renormalizable terms are taken into account many directions are completely flat. Non-renormalizable terms and soft symmetry breaking terms in a more realistic model lift the flat direction in a small amount. Certain fields can have large expectation values along these directions, a fact which is used in the Affleck-Dine mechanism for baryogenesis.

The space of all flat directions is called the moduli space and the field parameterizing the direction is called the modulus. This is a chiral superfield as given in eqn. (A.1), although the interesting component of it is just the
complex scalar field. This means a flat direction carries a global U(1) quantum number which can be baryon or lepton number. The vanishing of the potential is equivalent to the vanishing of the $D^a$ and $F$ auxiliary fields. As an example [59] consider the first Higgs and slepton doublet to obtain an expectation value,

$$H_1 = \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad L_1 = \begin{pmatrix} v \\ 0 \end{pmatrix}.$$  \hspace{1cm} (A.13)

Here $v$ is the VEV of the modulus field $\Phi$ parameterizing this particular flat direction. The MSSM superpotential of eqn. (A.12) does not contain the possible problematic term $H_1 L$, since this coupling is prohibited by R-parity. This means the F term is almost trivially zero. For the hypercharge, it is easily seen the $D^1$ term vanishes, since,

$$D^1 = g^2 (|H_1|^2 - |L|^2) = 0.$$  \hspace{1cm} (A.14)

In case of SU(2), the definitions of the Pauli matrices can be used or the D term can be written matrix valued as,

$$(D)_{ij} = D^a (T^a)_{ij}.$$  \hspace{1cm} (A.15)

Using the identity, valid for all SU(N),

$$(T^a)^i_j (T^a)^k_l = \delta^i_k \delta^j_l - \frac{1}{N} \delta^i_l \delta^j_k,$$  \hspace{1cm} (A.16)

we can write for the case at hand,

$$(D)_{ij} = \phi^i \phi^j - \frac{1}{2} |\phi|^2 \delta^i_j$$

$$= \begin{pmatrix} |v|^2 & 0 \\ 0 & |v|^2 \end{pmatrix} - \frac{1}{2} |v|^2 \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = 0.$$  \hspace{1cm} (A.17)

The same procedure can be done for SU(3). The interesting fact for baryogenesis is that the field carries a lepton number, a property very common in flat directions. As already pointed out, non-renormalizable terms and soft symmetry breaking terms can 'lift' this potential. This means the potential does not any more exactly vanish but contains small terms. This effects the vacuum expectation value of the modulus, but these higher order terms also often violate baryon and lepton number conservation. The first non-renormalizable term appearing in the superpotential of the model described above would be $\frac{1}{M^2} (H_1 L)^2$, giving rise to the potential,

$$V_{lift} = \frac{\Phi^6}{M^2}.$$  \hspace{1cm} (A.18)

Soft symmetry breaking terms are often added to the theory since otherwise supersymmetry would be unbroken and particles would be degenerated with
their super partners. This explicit way of breaking the symmetry does not affect the good properties of the model, hence the name soft. Included in these terms are for example mass terms for squarks and sleptons, Majorana mass terms for gauginos and cubic couplings of the scalar fields.
Acknowledgements

Above all I would like to thank my supervisor Tomislav Prokopec. I am very grateful for the many, many hours he invested in my research. He was always available for discussions and did not bother to make them extensive when there was much to talk about. At times I was confused he lighted the path of proceeding. Furthermore I would like to thank my parents and my brother. They kept believing in me and helped me at times it was hard to see the end of the road. I am also thankful to all master students in the students room 301 that helped me out when I was in conflict with Latex, mathematical problems, physical insights or anything else. Many thanks goes to all my friends on the faculty and otherwise, housemates of the Ridderschapstraat, colleagues of Vredenburg and family for supporting me during this period in my life. Finally, I would like to thank the cabins of the Minnaert, the two heatwaves of the summer of 2006, V&D for providing pencil and paper, Tosti† and Sokkie for keeping company and the bread toaster for being such a wonderful invention.
Bibliography


BIBLIOGRAPHY


